King Fahd University Of Petroleum and Minerals

College of Sciences

Mathematics Department

Math 101

Major Exam II

Section 19

Name:.......................................... ID#:.............. Ser#.................

NO CALCULATOR IS ALLOWED IN THE EXAM

SHOW ALL NECESSARY WORK

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Instructor: H. Al-Attas
1. Let \( f(x) = x^3 - 1 \) and let \( x_0 = 1 \) and \( x_1 = 2 \).

   (a) (3 points) Find the average rate of change of \( y \) with respect to \( x \) over the interval \([x_0, x_1]\).

   (b) (1 point) Find the slope of the secant line joining the points \((x_0, f(x_0))\) and \((x_1, f(x_1))\).

   (c) (5 points) Find the instantaneous rate of change of \( y \) with respect to \( x \) at the given value \( x_0 \). (Hint \( x^3 - y^3 = (x - y)(x^2 + xy + y^2) \))
(d) (1 point) Find the slope of the graph of $f$ at $x_0$.

(e) (5 points) Find the instantaneous rate of change of $y$ with respect to $x$ at a general $x$-value $x_0$.

(f) (3 points) Find the slope of the tangent line to the curve at $x_0 = 3$. (Hint: Use part (e))
2. (3 points) Given that the tangent line to \( y = f(x) \) at the point \((-1, 3)\) passes through the point \((0, 4)\), find \( f'(-1) \).

3. (5 points) Show that \( y = x^3 + 3x + 1 \) satisfies \( y''' + xy'' - 2y' = 0 \).

4. (6 points) Show that \( f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ x + 2, & x > 1 \end{cases} \) is not differentiable at \( x = 1 \).
5. (6 points) Answer TRUE or FALSE (Explain your answer by an example if it is false)
   Let \( y = f(x) \)
   (a) If \( f \) is continuous at \( x_0 \), then \( f \) is differentiable at \( x_0 \).
   (b) If \( f^{-1} \) is a continuous function, then \( f \) is a continuous function.

6. (5 points) Find \( \frac{dy}{dx} \) |\( x=1 \) where \( y = (2x^7 - x^2) \left( \frac{x-1}{x+1} \right) \).
7. (6 points) Find \( \frac{d}{dx} \cos x \).

8. (6 points) Determine where \( f(x) = \frac{1}{\sin x \cos x} \) is differentiable.
9. (9 points) Find \( \frac{d^2 y}{dx^2} \) if \( y = x \cos (5x) - \sin^2 x \).

10. (8 points) Find the equation of the tangent line to the curve \( x^3 + y^3 = 3xy \) at the point \( \left( \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \right) \).
11. (7 points) A 13-ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot be moving away from the wall when the top is 5 ft above the ground?

12. (8 points) Estimate the value of $\sqrt{36.3}$. 
13. (7 points) Let \( y = \frac{1}{x} \). Find \( dy \) and \( \Delta y \) at \( x = 1 \) with \( dx = \Delta x = -0.1 \).

14. The side of a square is measured to be 10 ft, with a possible error of \( \pm 0.1 \) ft.

   (a) (3 points) Use differentials to estimate the error in the calculated area.

   (b) (6 points) Estimate the percentage errors in the side and the area.
15. Let \( f(x) = 2x^5 + x^3 + 1 \)

(a) (5 points) Show that \( f^{-1} \) is differentiable on the interval \((-\infty, \infty)\).

(b) (2 points) Find a formula for the derivative of \( f^{-1} \).

16. Find the following limits

(a) (5 points) \( \lim_{x \to +\infty} \left( 1 + \frac{3}{x} \right)^{2x} \).

(b) (5 points) \( \lim_{x \to +\infty} \left( \frac{1}{\ln x} + 3e^{-2x+1} - 5 \right) \).
17. (Bonus) Find the values of $x$ at which the curve $y = \sqrt{3x + 1} (x - 1)^2$ has a horizontal tangent line.