King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester I, 2004-2005(041)
MATH 201
Final Exam
Date: January 6, 2005
Time: 3 hours

Student Name: ______________________________

Student Number: ______________________________

Section #: ______________________________

Serial #: ______________________________

Note:

- FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.
- Use of any calculator not allowed.

1. (a) Find equation of the surface obtained by rotating the curve (in the zx-plane) $z^2 - x^2 = 1$ about the x-axis. Draw a rough sketch of the surface. (5 points)

   (b) Draw the plane $x + y + z = 5$. (5 points)

   (c) Draw the plane $x + y = 2$. (2 points)

   (d) Draw the surface $y = x^2$. (3 points)

2. Fill in the following table:

Equation of Surface in 3 dimensions	Mathemaitcal Name	Sketch of surface
$z = y^2 + 2$		
$x^2 - 4y^2 = 4z^2$		
$z/2 = x^2 + y^2$		
$x^2 + y^2 - z^2 = 1$		

(10 points)

3. (a) A line L has parametric equations $x = 1 + t, \quad y = 2 + t, \quad z = 3t$. Let P be the point with coordinates $(2, 2, 0)$. Find a point Q on the line so that $\overrightarrow{PQ}$ is perpendicular to the line L. (5 points)

   (b) Find parametric equations of the line of intersection of the planes

$$-2x + 3y + 7z + 2 = 0$$
$$x + 2y - 3z + 5 = 0$$

4. (a) Find all points on the surface $x^2 + y^2 + 2z^2 = 1$ at which the tangent plane is parallel to the plane $2x + 2y + 8z = 1$. (5 points)

(b) Let $z = f(x - y, y - x)$. Show that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$. (5 points)

(c) Show, by using the definition of differentiability, that the function $f(x, y) = x^2 + y^2$ is differentiable at any point (x, y). (5 points)

5. Find the absolute maxima and minima of $f(x, y) = xy - x - 3y$ on the region inside and on the triangle with vertices $(0, 0)$, $(0, 4)$ and $(5, 0)$. (15 points)

6. (a) Evaluate the integral by changing to polar coordinates $\int_0^4 \int_0^{\sqrt{16-x^2}} \frac{dy\, dx}{(1+x^2+y^2)^{3/2}}$.
(10 points)

(b) Describe the region inside the circle $x^2 + y^2 = 4$ and to the right of the line $x = 1$ in polar coordinates. (5 points)

7. (a) Describe the solid below the paraboloid $z = x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 4$ by inequalities. (5 points)

(b) Find the volume of the solid in part (a). (10 points)

8. (a) Find the projection in xy-plane of the solid in the first octant below the plane $x/2 + z/2 = 1$ which is also bounded by the plane $y = 2$. (10 points)

   (b) Use (a) to draw the solid in part (a). (5 points)

9. (a) Describe the solid inside the sphere $x^2 + y^2 + z^2 = 2$ and above the plane $z = 1$ in spherical coordinates. [Give the range for θ and ϕ and for any such θ and ϕ the range for ρ]. (10 points)

   (b) Evaluate using spherical coordinates the volume of the solid in part (a). (5 points)

(The volume element dV in spherical coordinates is $dV = \rho^2 \sin\varphi \; d\rho \; d\varphi \; d\theta$)