King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

MATH 201

Final Examination

Semester I, 2004-2005(041)

January 6,2005

Time: 7:30a.m - 10:30a.m

Name:____ Section#:__________

ID#:____ Serial#:___________

Sec.#	Instructor	Location
08,11	Dr. Abdul Rahim Khan	M.P.H Building

Instructions:

1. Do not use programmable calculators. Use of ordinary calculator is allowed.
2. Show all your work. Less credit will be given for answer not supported by proper work.
3. Clearly indicate the theorem or result you use.
4. This exam consists of 13 pages.
5. Do not forget to write your NAME, ID#, Section # and SERIAL# in the space provided above.

Question#	Grade/Points
1	______________/18
2	______________/18
3	______________/18
4	______________/18
5	______________/18
6	______________/20
Total	______________/110

1. (a) Find polar coordinates of all points at which the curve $r = 5\sin\theta$ has a horizontal or a vertical tangent line.

(9 Points)

(b) Find area of the region inside the cardioid $r = 2 + 2\cos\theta$ and to the right of the line $r\cos\theta = \dfrac{3}{2}$.

(9 Points)

2. (a) Suppose two lines have parametric equations:

$$l: \quad x = 2t + 1, \quad y = -t + 3, \quad z = 5t$$
$$m: \quad x = 3 + 4t, \quad y = 2 - t, \quad z = 2t$$

(i) Find distance from $A(3, 1, -1)$ to l.

(ii) Do the lines l and m intersect ? If yes, find their point of intersection.

(9 Points)

(b) The graph of $9x^2 + 4y^2 = 36$ is revolved about the y-axis. Find the equation of the resulting surface S. Identify the surface S and give a rough sketch of this surface.

(9 Points)

3. (a) Check whether or not $U(x,y) = \ln(x^2+y^2), V(x,y) = 2\tan^{-1}\left(\frac{y}{x}\right)$ satisfy the Cauchy - Riemann equations.

(5 Points)

(b) Let f be a differentiable function of one variable. Assume that $z = f(x^2+y^2)$. Use the chain rule to calculate $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y}$.

(4 Points)

(c) Let $f(x, y) = \dfrac{x^3 y}{x^2 + y^2}$.
Find the degree n of the homogeneous function $f(x, y)$. Verify the formula $x f_x(x, y) + y f_y(x, y) = n f(x, y)$

(9 Points)

4. (a) Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $x^2 + 4y^2 + z^2 = 4$ at the point $(1, -1, 2)$.

(9 Points)

(b) Examine the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ for the local extrema.

(9 Points)

5. (a) Use the Lagrange multipliers to find the extrema of $f(x, y, z) = x + 2y - 3z$ subject to the constraint $z = 4x^2 + y^2$.

(9 Points)

(b) Evaluate $\int_0^1 \int_y^1 \sin x^2 \, dx \, dy$

(5 Points)

(c) Set up an iterated triple integral to find volume of solid bounded by the cylinder $z = x^2$ and the planes $y = 0$ and $y + z = 7$.

(4 Points)

6. (a) Use polar coordinates to evaluate the integral $\iint\limits_R (x + y)dA$ where R is the region bounded by the circle $x^2 + y^2 = 2y$.

(10 Points)

(b) Use spherical coordinates to find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and below by the cone $z = \sqrt{x^2 + y^2}$.

(10 Points)