

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2004-2005(042)
MATH 202
Major Exam 1
Date: March 14, 2005

Student Name: _____

Student ID: _____

Section: _____

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. Find all solutions—including singular solutions—of the differential equation

$$xy' - 3y = 0.$$

2. (a) Verify that $x = c_1 \cos t + c_2 \sin t$ is a 2-parameter family of solutions of $x'' + x = 0$.
- (b) Find a solution of $x'' + x = 0$ with initial conditions $x(0) = -1$, $x'(0) = 8$.

3. Verify that $y = e^{-x^2} \int_0^x e^{t^2} dt + c_1 e^{-x^2}$ is the general solution of the differential equation

$$\frac{dy}{dx} + 2xy = 1.$$

4. Solve $\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$ by separating variables [Hint: First, factorize the numerator and the denominator]

5. Solve the initial value problem

$$x^2 \frac{dy}{dx} = y - xy, \quad y(-1) = -1$$

by separating the variables.

6. Find a continuous solution of the differential equation

$$\frac{dy}{dx} + 2y = f(x)$$

where $f(x) = \begin{cases} 1, & 0 \leq x \leq 3 \\ 0, & x > 3 \end{cases}$ with initial condition $y(0) = 0$.

7. (a) Find an integrating factor for the equation

$$(x^2 + y^2 - 5)dx = (y + xy)dy.$$

- (b) Use (a) to solve

$$(x^2 + y^2 - 5)dx = (y + xy)dy$$

with initial condition $y(0) = 1$.

8. Recall: Two curves are perpendicular at a point of intersection P if their tangent lines are perpendicular at P . Find a family of curves which is perpendicular to the family of hyperbolas $x^2 - y^2 = c$ at every point of intersection.