King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2004-2005(042)
MATH 202
Final Exam
Date: June 8, 2005

Student Name: ______________________________

Student ID: ______________________________

Section: ______________________________

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. (a) Define ordinary and singular points of the differential equation

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0.$$

(b) Find singular points of

$$(\sin x \cos x)y'' + xy' + (\sin x)y = 0.$$

(c) The equation in (b) has a power series solution $\sum_{n=0}^{\infty} a_n\, x^n$. If R is the radius of convergence of this series, then R is $\geq$ __________________ (15 points)

2. A function y satisfies $y''(x) + (\tan x)y' + (\sin x)y = 0$. If y has a power series solution $y = \sum_{n=0}^{\infty} a_n x^n$, and $y(0) = 1, \quad y'(0) = 2$, find a_0, a_1, a_2 and a_3 using the given differential equation. (10 points)

3. (a) Show that $x_0 = 0$ is a regular singular point of $xy'' - xy' - y = 0$.

   (b) Verify that the indicial equation of the equation in (a) is $r(r-1) = 0$.

   (c) Verify that if $\sum_{n=0}^{\infty} a_n x^{n+1}$ is a solution of $xy'' - xy' - y = 0$, then

   $$a_k = \frac{a_{k-1}}{k}, \quad k = 1, 2, \ldots.$$

   (d) Find a_k in terms of a_0 and k.

(30 points)

4. Solve the system of differential equation

$$\frac{dx}{dt} = 2x + 2y$$

$$\frac{dy}{dt} = 2y$$

using the method of eigenvalues and eigenvectors. (20 points)

5. Find annihilators of lowest order of

   (a) $(1+e^x)^2$. (5 points)

   (b) $\sin^2 x - \cos^2 x$ (5 points)
   (Hint: Use a trigonometric identity).

6. Solve the initial value problem $\left(\dfrac{dy}{dx}\right)^2 = 1 - y^2, \quad y(0) = 1.$ (10 points)

7. Find all solutions – including singular solutions if any of $\dfrac{dy}{dx} = (y^2 - 1)$.

(10 points)

8. Check that

(a) $\dfrac{d^2}{dx^2} - 2\dfrac{d}{dx} + 1 = \left(\dfrac{d}{dx} - 1\right)\left(\dfrac{d}{dx} - 1\right).$

(b) By using the substitution $z = \left(\dfrac{d}{dx} - 1\right)(y)$, solve the equation

$$\left(\frac{d}{dx} - 1\right)\left(\frac{d}{dx} - 1\right)(y) = 0\,.$$

(10 points)

9. (a) Define linear independence of functions y_1 and y_2.

   (b) Show that if y_1 is a non-zero solution of $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$ (where $a_2(x) \neq 0$), and $y_2 = uy_1$ is a second solution, then $y_1,\ y_2$ are linearly independent. (10 points)

10. (a) Change the equation

$$(2x-1)^3y''' + (2x-1)^2y'' = 0$$

into a linear differential equation with constant coefficients.

(b) Solve the equation

$$x^3y''' - 6y = 0.$$

(10 points)