Name:__________________________________________
ID #:____________________Section #_________ Serial #:_____

• Write clearly.
• Show all your steps.
• No credit will be given to wrong steps.
• Do not do messy work.
• Calculators are not allowed in this exam.

Mark: _______

40
Problem 1 (6 points)
(i) Solve the IVP: \( y' + (\tan x)y = \cos^2 x, \quad y(0) = -1 \)
(ii) Find a one-parameter family of solutions of
\[ 2x \sqrt{9 - y^2} dx = dy \]
and specify one singular solution.
Problem 2 (4 points)
Find $k$ so that the differential equation $(6xy^3 + \cos y) \, dx + (2kx^2y^2 - x \sin y) \, dy = 0$ is exact. Then solve the equation.
Problem 3 (4 points)
Consider the differential equation \((1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0\).
(i) Verify that \(y_1 = x + 1\) is a solution
(ii) Find a second solution and the general solution.
Problem 4 (5 points)
Solve the IVP: \( 4x^2y'' + 17y = 0, \quad y(1) = -1, \quad y'(1) = 0 \)
Problem 5 (6 points)

(i) Write the power series representations for $\frac{1}{1+x^2}$ and $\frac{x}{1+x^2}$

(ii) Show that $x_0 = 0$ is an ordinary point of the d.e. $(x^2 + 1)y'' - 2xy' + 2y = 0$

(iii) Find two power series solutions of the above d.e. (Give few terms only).
Problem 6 (5 points)
(i) Show that $x_0 = 0$ is a regular singular point of the d.e. $xy'' - xy' + y = 0$.
(ii) Find two series solution about the regular singular point. (Give few terms only).
Problem 7 (5 points)
Consider the linear system \( X'(t) = AX(t) \) where \( A \) is a \( 2 \times 2 \) real constant matrix. What is the general solution of the system if it is known that \( \lambda_1 = 1+2i \) is an eigenvalue and \( K_1 = \begin{pmatrix} 1 \\ i \end{pmatrix} \) is the corresponding eigenvector.
Problem 8 (5 points)
Write the system \( \frac{dx}{dt} = 2x + y, \frac{dy}{dt} = -x \) in matrix form, and then solve the system.