Prob. 1
Calculate $\int_0^{\ln \sqrt{2}} \frac{1 + \cos(\frac{1}{2^x})}{e^{2x}} dx$
Prob. 2
Evaluate the following limit by interpreting it as a Riemann sum in which the given interval is divided into $n$ subintervals of equal width

$$
\lim_{n \to \infty} \sum_{k=1}^{n} \frac{\pi}{4n} \sec^2 \left( \frac{\pi k}{4n} \right); \quad [0, \pi/4].
$$
Prob. 3
(a) Give a geometric argument to show that
\[
\frac{1}{x + 1} < \int_{x}^{x+1} \frac{dt}{t} < \frac{1}{x}, \quad x > 0
\]
(b) Use the result in part (a) to prove that
\[
\frac{1}{x + 1} < \ln \left(1 + \frac{1}{x}\right) < \frac{1}{x}, \quad x > 0
\]
(c) Use the result in part (b) to prove that
\[
e^{\frac{1}{x+1}} < \left(1 + \frac{1}{x}\right)^{x} < e
\]
and hence that \(\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e\).
(d) Use the inequality in part (c) to prove that
\[
\left(1 + \frac{1}{x}\right)^{x} < e < \left(1 + \frac{1}{x}\right)^{x+1}, \quad x > 0.
\]
Prob. 4
Find the area of the region enclosed between the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$
and the coordinate axes.
Prob. 5
Use cylindrical shells to find the volume obtained by revolving the area in the first quadrant between \( x = 3y^3 - 4y^4 \) about the x-axis
Prob. 6
Find by two different methods the volume when the region bounded by
\( y = \frac{1}{3} \cos x \) and \( y = 3x^4 \) is revolved about the
a) \( x = 4 \)
b) \( y = 3 \)
c) \( x\)-axis
d) \( y\)-axis
Prob. 7
What is the nature of
(a) \[ \sum_{k=1}^{\infty} \left( \frac{k!}{k^r} \right)^k \]
(b) \[ \sum_{k=2}^{\infty} \frac{k}{(\ln k)^r} \]
Prob. 8
What is the nature of
(a) \( \sum_{k=1}^{\infty} \frac{(2k)!^2 2^k}{(5k+5)!} \), (b) \( \sum_{k=1}^{\infty} \frac{\sqrt{k} \ln \sqrt{k}}{k^4+1} \)
Prob. 9

Classify the series as absolutely convergent, conditionally convergent or divergent

(a) \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^3 + 2} \), (b) \( \sum_{k=1}^{\infty} \sin \frac{2k\pi}{3} \), (c) \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k+1} + \sqrt{k}} \), (d) \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1} (k+7)!}{(5k-1)!} \)