King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math - 260 Applied Calculus

Test No. 2 | Dr. M. Z. Abu-Sbeih | Aaugust 13, 2005

Serial No: | Student No.: | Name:

1. SHOW ALL WORK. NO CREDITS FOR ANSWERS NOT SUPPORTED BY WORK.
2. All TYPES OF CALCULATORS ARE NOT ALLOWED.

1. **(15 points)** Solve the system of equations the system:

$$x + 2z = 2$$
$$2y + z = 1$$
$$x - y + z = 2$$

by using either (a) inverse matrix, or (b) Cramer's Rule.

2. **(15 points)** Consider the vectors: $\vec{a} = (1\ 3\ 1), \vec{b} = (-1\ \ 0\ \ -2),$ and $\vec{c} = (2\ 1\ 1)$.

a) Determine whether the vectors are linearly dependent or independent.

b) Is it possible to write $\vec{a}$ as a linear combination of $\vec{b}$ *and* $\vec{c}$.

c) Write the vector $\vec{v} = (4\ \ 4\ \ 4)$ as a linear combination of $\vec{a}, \vec{b}$ *and* $\vec{c}$.

3. **(15 points)** Find a basis and the dimension of the solution space of the homogeneous system:

$$\begin{aligned} x_1 - x_2 - x_3 + 2x_4 + x_5 &= 0 \\ 2x_1 - x_2 + x_3 - 3x_4 + 2x_5 &= 0 \\ x_1 + x_2 - 2x_3 + 2x_4 + x_5 &= 0 \end{aligned}$$

4. **(15 points)** Find the general solution of : $(D^5 + 8D^3 + 16D)y = 0.$

5. **(17 points)** Use the method of undetermined coefficient to find the general solution of the equation: $y'' - 4y' + 3y = 2\cos x + 4\sin x.$

6. **(18 points)** Use the method of variation of parameters to solve the equation:
$y'' + y = \sec^3 x \tan x$.

7. **(20 points)** Consider the matrix $A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$

   a. Find the eigenvalues and the corresponding eigenvectors of *A*.
   b. Is the matrix *A* diagonalizable? If yes, find an invertible matrix *P* and a diagonal matrix *D* such that $P^{-1}AP = D$.
   c. Find A^5.