

Serial No:

Student No.:

Name:

1. SHOW ALL WORK. NO CREDITS FOR ANSWERS NOT SUPPORTED BY WORK.
2. ALL TYPES OF CALCULATORS ARE NOT ALLOWED.

Problem 1 (15 Points): Show that the equation is exact and solve. $3y(x^2 - 1) dx + (x^3 + 8y - 3x) dy$.

Problem 2 (15 Points): The general solution of the differential equation $y'' - 25y = 0$, is

$y = Ae^{5x} + Be^{-5x}$. Find the particular solution which satisfies the initial conditions: $y(0) = 10$ and $y'(0) = 20$.

Problem 3 (15 Points): Solve the DE: $\frac{dy}{dx} = -\frac{2x+3y-1}{2x+3y+2}$.

Problem 4 (15 Points): For a substance C, the time rate of conversion is proportional to the square of the amount x of unconverted substance present. If we start with 2 kg of the substance, and after one hour we found that one kg left, how long it will take to have only 1/2 kg left?

Problem 5 (10 Points): Determine whether each of the following statements is true or false.

- (a) For any matrix A , if $A^2 = \mathbf{0}$, then $A = \mathbf{0}$ (where $\mathbf{0}$ is the zero matrix).
- (b) For any matrix A , if $A^2 = \mathbf{I}$, then $A = \mathbf{I}$ (where \mathbf{I} is the identity matrix).
- (c) $A + B = B + A$ for any two matrices A and B .
- (d) If \bar{x}_1 and \bar{x}_2 are two solutions of the homogeneous system $A\bar{x} = \mathbf{0}$, then $2\bar{x}_1 - 3\bar{x}_2$ is also a solution.
- (e) For any two matrices A and B , if $AB = B$, then $A = \mathbf{I}$ (the identity matrix).

Problem 6 (15 Points): Solve the DE: $2x^3y' = y^3 + 3x^2y$.

Problem 7 (15 Points): Consider the linear system of equations:

$$x + y - z = 5$$

$$3x + y + 3z = 11$$

$$4x + y + 5z = 14$$

- (a) Write the augmented matrix of the system.
- (b) Use Gaussian elimination to write the matrix in echelon form.
- (c) Use Gauss-Jordan elimination to write the matrix in reduced echelon form.
- (d) Use part c to find the solution of the system.