Important Note

Show all work.
Use of programmable calculator is not allowed.
Mobiles and paging devices should not be carried during examination.

Instructor: F. D. Zaman
Q 1) Let \( F(x, y, z) = \ln \left( \frac{yz}{x} \right) \). Find the direction in which this function decreases most rapidly at \( P(1,2,1) \).
Q 2) Show that the integral \( \int_{C} (yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}) \cdot d\mathbf{r} \) is independent of path.

Hence find its value along any curve \( C \) from \( A(0,0,0) \) to \( B(3,1,4) \). (4)
Q 3) Use Green’s theorem to evaluate \( \oint_C (y^2 + e^x) \, dx + (x^2 - \cos y) \, dy \)

Where \( C \) is the triangle in anti-clockwise direction having vertices A(0,0), B(1,0) and C(0,1). (4)
Q 4) Write the flux integral \( \iint_S F \cdot n \, ds \) where \( F = x^2 i + y^2 j + 2zk \) and S is the portion of paraboloid \( z = 4 - x^2 - y^2, \; 0 \leq z \leq 4 \). Change this integral into an integral over region R in suitable coordinates. DO NOT EVALUATE THIS INTEGRAL. (3)
Q 5) Use Stokes’ theorem to write \( \oint_C \mathbf{F} \cdot d\mathbf{r} \) into an integral over surface \( S \). Then change the surface integral into a double integral over region formed by \( S \).

\[
\mathbf{F} = z^2y \cos x \mathbf{i} + z^2x(1 + \sin xy) \mathbf{j} + 2ze^{xy} \mathbf{k}
\]

\( C: \text{formed by plane } x + y + z = 1, \text{ and coordinate planes.} \)

DO NOT EVALUATE THE INTEGRAL. (3)
Q 6) (a) Find $L\{t \sin t\}$

(b) Find $L^{-1}\left\{ \frac{s}{(s+2)(s^2+4)} \right\}$