King Fahd Univ. of Petroleum and Minerals
Faculty of Sciences
Department of Mathematical Sciences

FINAL EXAM
(MATH. 101-051 Section 18)

Name:
ID:

Important instructions:
- Use an HB pencil or a pen (do not use red color)
- Solve the problems completely
- Write down your answers in a clear manner
- Justify all your steps
- Use the back of the page (verso) only for scratching
Prob. 1
Find the following limit

\[ \lim_{v \to \pi/4} \frac{v - \pi/4}{\tan v - 1} \]
Prob. 2

Show that the triangle that is formed by any tangent line to the graph of $y = 1/x$, $x > 0$, and the coordinate axes has an area of 2 square units.
Prob. 3
Suppose that a function $f$ is differentiable at $x = 3$ and \( \lim_{h \to 0} \frac{f(3+h)}{h} = -4 \), find $f(3)$ and $f'(3)$. 
Prob. 4
An open box is to be made from a 3-ft by 8-ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume that the box can have.
Prob. 5
Use the Mean-Value Theorem to prove that

\[ \frac{x}{1 + x^2} < \tan^{-1} x < x, \quad (x > 0). \]
Prob. 6
In each part, find all critical numbers, and use the first derivative test to classify them as relative maxima, relative minima, or neither
(a) \( f(x) = x^{1/3}(x - 7)^2 \)
(b) \( g(x) = 2 \sin x - \cos 2x \)
(c) \( h(x) = 3x - (x - 1)^{3/2}. \)
Prob. 7
Analyse (completely) and sketch the graph of $f(x) = \tan(x^2 + 1)$