King Fahd University Of Petroleum and Minerals
College of Sciences
Mathematics Department
Math 102
Major Exam I
Section 8

Name:............................  ID#:.............  Serial #:.............

NO CALCULATOR IS ALLOWED IN THE EXAM

SHOW ALL NECESSARY WORK

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Instructor: H. Al-Attas
1. (6 points) Find a function $f$ such that $f''(x) = x + \cos x$ and such that $f(0) = 1$ and $f'(0) = 2$.

2. (5 points) Show that $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$.
3. (8 points) Use subintervals of equal lengths and use $x_k^*$ as the left endpoint of each subinterval to find the net signed area under the curve $f(x) = 1 - x^2$ and over the interval $[0, 3]$.

4. (5 points) Given $\int_1^3 f(x) \, dx = \alpha$ where $f(x) = \sqrt{1 + x^2}$ and $\alpha > 3$.

   (a) Find $f_{\text{ave}}$ in terms of $\alpha$

   (b) Find all values $x^*$ (in terms of $\alpha$) that satisfy the Mean Value Theorem for Integrals.
5. (5 points) If \( F(x) = \int_{x^2}^{3x^2} \sin^2 \frac{1}{t+1} \, dt \), then find \( F'\left(\sqrt{\frac{2}{\pi}}\right) \).

6. (5 points) Find the following limits

(a) \( \lim_{x \to 0} (1 + 3x)^{\frac{2}{x}} \)

(b) \( \lim_{n \to +\infty} \left( \frac{3+6+9+12+\ldots+n}{n^2} + \sum_{k=1}^{n} \frac{2k^2}{n^2} \right) \).
7. (5 points) Solve the equation \( \sum_{k=4}^{n-2} k = 624 \) where \( n \) is a positive integer.

8. (6 points) Evaluate the integral \( \int_0^3 (1 + \sqrt{6x - x^2}) \, dx \) by using a "Geometry Formula". (Hint: Sketch the region) points)
9. (10 points) Evaluate the integral $\int_0^4 \sqrt{x} \, dx$ by using the definition of Riemann integration. Use subintervals of unequal lengths given by the partition

$$0 < \frac{4(1)^2}{n^2} < \frac{4(2)^2}{n^2} < \frac{4(3)^2}{n^2} < \ldots < \frac{4(n-1)^2}{n^2} < 4$$

and let $x^*_k$ be the right end point of the $k$th subinterval.
10. (5 points each) Evaluate the following integrals:

(a) \[ \int \frac{x^2}{4+x^6} \, dx \]

(b) \[ \int_0^1 x (1-x)^n \, dx \] where \( n \) is a positive integer
(c) \( \int \frac{x + x^3}{1 + x^2} \, dx \)

(d) \( \int_1^4 |3x - 6| \, dx \)
(e) \[ \int_{1}^{3} \frac{dx}{\sqrt{x(1+x)}} \]

(f) (Bonus) \[ \int \frac{dx}{9x^2 + 12x + 5} \]