KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

Math 102- Term 051-I
Final Exam
Sunday, January 29, 2006
Section 12

Instructor: Marwan Al-Momani **Duration: 180 minutes**

Name: ID# Serial:

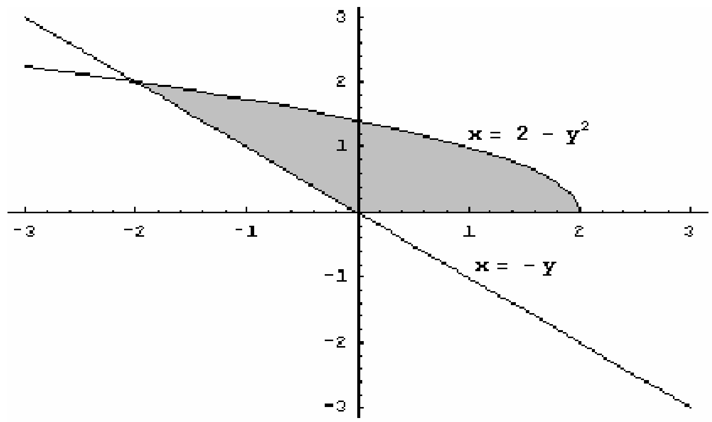
Note: **Using Calculators is not allowed in this Exam**

Question No	Full Marks	Marks Obtained
Part I: Multiple Choice	**30**	
Part II: Written		
Q1	**16**	
Q2	**22**	
Q3	**18**	
Q4	**20**	
Q5	**16**	
Q6	**8**	
Total	**130**	

Question Number	***Correct Choice***				
Q1	a	b	c	d	e
Q2	a	b	c	d	e
Q3	a	b	c	d	e
Q4	a	b	c	d	e
Q5	a	b	c	d	e
Q6	a	b	c	d	e
Q7	a	b	c	d	e
Q8	a	b	c	d	e
Q9	a	b	c	d	e
Q10	a	b	c	d	e
Q11	a	b	c	d	e
Q12	a	b	c	d	e

<u>Part I:</u> Multiple Choice Questions
Choose the correct answer (2.5-Points each = <u>30 Points</u>)

1. The area of the shaded region of the given graph equals to:
   a. $\frac{14}{3}$
   b. $\frac{5}{3}$
   c. $\frac{10}{3}$
   d. $\frac{8}{3}$
   e. $\frac{16}{3}$



2. If $f(x)$ is continuous function such that : $\int_{\frac{\pi}{8}}^{x} t\, f(t)\, dt = \sin(x) - x\cos(x)$, then $f\left(\frac{\pi}{3}\right) = ?$
   a. $\frac{\sqrt{3}\,\pi}{6}$
   b. $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$
   c. $\frac{\pi}{3}$
   d. $\frac{\sqrt{3}}{2}$
   e. 0

3. The value of : $\lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \frac{\sec^2\left(x_k^*\right)}{\sqrt{1-\tan^2\left(x_k^*\right)}}$ over the interval $\left[0\ ,\ \frac{\pi}{4}\right]$ equals to:

   a. 0
   b. $\frac{\pi}{2}$
   c. $\frac{\pi}{4}$
   d. π
   e. $-\frac{\pi}{2}$

4. The volume of the solid generated when the region enclosed by $y=\tan(x)$, $y=1$ and $x=0$ is revolved about the $x-axis$ equals to:

   a. $\int_0^{\frac{\pi}{4}}\left(1-\tan(x)\right)dx$
   b. $\pi\int_0^{\frac{\pi}{4}}\left(\tan(x)-1\right)dx$
   c. $\pi\int_0^{\frac{\pi}{4}}\left(2-\tan^2(x)\right)dx$
   d. $\pi\int_0^{\frac{\pi}{4}}\left(2-\sec^2(x)\right)dx$
   e. $\pi\int_0^{\frac{\pi}{4}}\sec^2(x)\,dx$

5. The exact arc length of the parametric curve given by :
   $x(t) = \cos(2t), y(t) = \sin(2t), 0 \le t \le \frac{\pi}{2}$ equals to:
   a. $\frac{\pi}{2}$
   b. $\frac{3\pi}{2}$
   c. π
   d. 4π
   e. 2π

6. $\int \sec h(\ln(x))dx = ?$
   a. $\ln(2x+1)+c$
   b. $\ln(x^2+1)$
   c. $\ln(\ln(x))+c$
   d. $\ln\left|\sec h(\ln(x)) + \tan(\ln(x))\right| + c$
   e. $\ln(x^2+1)+c$

7. If $\sinh(x) = \frac{2}{5} - \cosh(x)$, then $x = ?$

a. $\ln\left(\frac{5}{2}\right)$

b. $-\ln(5)$

c. $\ln(3)$

d. $\ln\left(\frac{2}{5}\right)$

e. $\ln(10)$

8. The following partial fraction decomposition :

$\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{(x+1)^4} + \frac{Ex+F}{(x^2+5)}$ is for :

a. $\frac{1}{(x+1)(x^2+5)}$

b. $\frac{x}{(x^2+2x+1)(x^2+5)}$

c. $\frac{x}{(x^2+2x+1)^2(x^2+5)}$

d. $\frac{1}{(x+1)^2(x^2+5)^2}$

e. None of the above.

9. The value of $\int_2^{17} \frac{1}{\sqrt{x-1}+\sqrt[4]{x-1}}\,dx = ?$

   a. $4\ln\left(\frac{3}{2}\right)$
   b. $2+\ln\left(\frac{81}{16}\right)$
   c. $4+2\ln\left(\frac{3}{2}\right)$
   d. $2+4\ln\left(\frac{2}{3}\right)$
   e. 4

10. The sequence $\left\{n^{\frac{1}{n}}\right\}_{n=1}^{\infty}$ is:

   a. Not monotone
   b. Eventually increasing
   c. Increasing
   d. Eventually decreasing
   e. Decreasing

11. The sum of the series $\sum_{k=2}^{\infty} \frac{1-2^k}{5^k}$ equals to:

a. $-\frac{13}{60}$

b. $-\frac{4}{15}$

c. $\frac{4}{15}$

d. $\frac{13}{60}$

e. $\frac{1}{20}$

12. Which one of the following statements is TRUE?

a. If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ is convergent

b. If $\sum |a_k|$ is divergent, then $\sum a_k$ is divergent.

c. If $a_k \le b_k$ for all $k \ge 1$, and $\sum a_k$ converges, then $\sum b_k$ converges.

d. If $\int_1^{\infty} \frac{\tan^{-1}(x)}{1+x^2}dx$ converges to $\frac{3\pi^2}{32}$ then $\sum_{k=1}^{\infty} \frac{\tan^{-1}(k)}{1+k^2}$ converges to $\frac{3\pi^2}{32}$.

e. The sequence $\left\{n e^{-2n}\right\}_{n=1}^{\infty}$ is strictly decreasing.

<u>Part II:</u> Written Questions.

<u>Provide neat and complete solutions to each question. Show necessary steps for full credit.</u>

<u>Question One</u>: (5+5+6=<u>16-Points</u>)

Evaluate the following integrals:

a. $\int \tan^5(x)\sec^3(x)\,dx$

b. $\int \frac{1}{\sqrt{9+x^2}}\,dx$

c. $\int_3^\infty \frac{1}{x(x+1)}\,dx$ If converges.

Question Two (6+6+4+6=22-Points)

Use any method to determine whether the following series converge or diverge.

a. $\displaystyle\sum_{k=3}^{\infty}\frac{1}{k}\left(\frac{1}{\ln(k)}\right)^{1/2}$

b. $\displaystyle\sum_{k=2}^{\infty}\frac{(k-3)(k+4)}{k(k+1)(k-1)}$ (Hint: Use limit comparison test)

c. $\sum_{k=1}^{\infty}\left(2+e^{-2k}\right)^{k}$

d. $\sum_{k=1}^{\infty}\frac{(k!)^{2}3^{k}}{(2k)!}$

<u>Question Three (4+3+8 = 18-Points)</u>

Let $\{a_n\}$ be a sequence defined recursively by: $a_1 = 1,\ a_{n+1} = \sqrt{6 + a_n}\ , n \geq 1$

a. Show that $a_n \leq 3$ for all $n \geq 1$

b. Show that $a_{n+1}^2 - a_n^2 = (3 - a_n)(2 + a_n)$

c. Use the results in part (a) and (b) to **show that** $\{a_n\}$ **converges**, and **find the limit** L

Question Four (12+8 = 20-Points)

a. Determine whether the following series is absolutely convergent, or conditionally convergent or divergent. $\sum_{k=2}^{\infty} (-1)^k \frac{k^2+1}{k^3+2}$

b. Find the interval and the radius of convergence of $\sum_{k=1}^{\infty} (-1)^k \frac{(x+2)^k}{k^2 3^k}$

Question Five(9+7 = 16-Points)

Find the Taylor series and use sigma notation to write it for :

a. $\sin(\pi x)$ about $x_0 = \frac{1}{2}$

b. $\dfrac{x^2}{(1+x)^3}$ about $x_0 = 0$ (Hint: Use the **binomial series:**

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots + \frac{m(m-1)\cdots(m-k+1)}{k!}x^k + \cdots$$

Question Six (4+4=8-Points)

a. Integrate the Maclaurin series $\frac{1}{1-x}$ for and use the result to show that

$\sum_{k=1}^{\infty} \frac{x^k}{k} = -\ln(1-x)$ (Hint: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$)

b. Use the result in part (a) to find the sum: $\sum_{k=1}^{\infty} \frac{1}{k\,4^k} = \frac{1}{4} + \frac{1}{2(4^2)} + \frac{1}{3(4^3)} + \cdots$