1. (23 points) Consider the function: 
\[ f(x) = \frac{1}{x^2 + 1} \]
with 
\[ f'(x) = \frac{-2x}{(x^2 + 1)^2} \]
and 
\[ f''(x) = \frac{2(3x^2 - 1)}{(x^2 + 1)^3}. \]

a) Find all vertical and horizontal asymptotes.

b) Find the critical numbers.

c) Find the increasing and decreasing intervals.

d) Find the local extrema of \( f(x) \).

e) Find the concavity intervals and the inflection points if any exists.

f) Sketch the graph of the function. **Clearly indicate all important points on the graph**, such as, extrema, inflection points, and intercepts if any such points exist. Also the concavity must be clear.
2. (9 points) A cylindrical can, open at the top, is to be made from a fixed amount of material, \(300\, \text{cm}^2\). Find the radius and height of the can which will give maximum volume.

3. (9 points) The demand equation for a product is \(p = \frac{10}{\sqrt{q}}\). Use differentials to approximate the revenue \(R(q)\) when 26 units are sold.

4. (9 points) The marginal cost function is given by \(C'(q) = \frac{q}{1+q^2}\). If the fixed cost is 2000 SR, find the total cost when 10 units are produced.
5. (32 points) Evaluate the following integrals:
   
   a. \[ \int \left( \frac{1}{(x-1)^3} + \frac{1}{x-1} \right) \, dx \]
   
   b. \[ \int \frac{2^x}{1+2^x} \, dx \]
   
   c. \[ \int \frac{(x+1)^2}{\sqrt{x}} \, dx \]
6. (9 points) Find the area between the graph of $y = 4 - x^2$ and the $x$-axis from $x = 0$ to $x = 3$.

7. (9 points) Find the area enclosed by the graphs of $y = x^2 - x$ and $y = x + 3$.