# King Fahd University of Petroleum & Minerals

Department of Mathematical Sciences

**Math 201(Calculus III)(051)**

**Major Examination II (Sec # 15)**

**December 12, 2005**

**Time: 90 Minutes** **Marks:…../60**

Name: ______________________ Serial #: __________

ID#: ______________________ Section #: __________

(1) Show complete and neat work for full credit.

(2) This exam consists of (9) pages.

Q 1. Find distance between the lines

$L_1 : x = 3 + t, y = 2 - 4t, z = t$

$L_2 : x = 4 - t, y = 3 + t, z = -2 + 3t.$

(10 Points)

Q. 2 Find equation of a plane that contains the line $L:\ x = 1 + 3t, y = 3 + 2t, z = 4t$ and is parallel to the intersection of the planes (5 Points)

$$2x - y + z = 0$$
$$y + z + 1 = 0$$

Q.3 (a) Find equation of surface of revolution by revolving the curve $x^{2/3} + y^{2/3} = 1$ about the $x - axis$. (5 Points)

(b) Describe and sketch curve of intersection between the surfaces

$z = x^2 + y^2$ and $z = 10 - x^2 - y^2$. (5 Points)

Q.4. Evaluate:

$$\lim_{(x,y)\to(0,0)}\left[\frac{x^2-y^2}{\sqrt{x^2+y^2}}+\frac{\sin h(4x^2+4y^2)}{x^2+y^2}\right]$$

(5 Points)

Q. 5 Let $U(x,y) = \ln(x^2 + y^2)$

$$V(x,y) = 2\tan^{-1}\left(y/x\right).$$

Show that $U(x,y)$ and $V(x,y)$ satisfy Cauchy-Riemann equations. (5 Points)

Q.6 Let $f(x,y,z)=\begin{cases}\dfrac{xyz}{x^3+y^3+z^3} & (x,y,z)\neq(0,0,0)\\ 0 & (x,y,z)=(0,0,0)\end{cases}$ (10 Points)

(i) Calculate $f_x(0,0,0)$.

(ii) Is $f$ continuous at $(0,0,0)$ ?

(iii) Is $f$ differentiable at $(0,0,0)$ ?

Q. 7 (a) Let $w = r^2 + sv + t^3$ where $r = x^2 + y^2 + z^2, s = xyz, V = xe^y$ and $t = yz^2$.
Use chain rule to find $\dfrac{\partial w}{\partial z}$. (5 Points)

(b) Find a unit vector in whose direction $f(x, y) = x^2 e^{-2y}$ decreases most rapidly at $P(2, 0)$. Find the rate of change of $f$ at $P$ in that direction. (5 Points)

Q. 8. Find the point on the paraboloid $z = 9x^2 + 4y^2$ at which the normal line is parallel to the line through $P(-2, 3, 5)$ and $Q(6, -1, 1)$. (5 Points)