King Fahd University of Petroleum & Minerals

Department of Mathematical Sciences

Math 201, Section: 15

Final Examination

Semester I, 2005-2006 (051)

January 25, 2006

Time: 7:30 A.M to 10:30 A.M

Name:____ Section #:__________

ID#: ____ Serial #: __________

Sect #	Instructor	Location
15	Dr. Abdul Rahim Khan	Building 10 (Auditorium)

Instructions:

1. Do not use programmable calculators. Use of ordinary calculator is allowed.
2. Show all your work. Less credit will be given for answer not supported by proper work.
3. Clearly indicate the theorem or result you use.
4. This exam consists of 13 pages.
5. Do not forget to write your NAME, ID#, Section# and Serial# in the space provided above.

Question #	Grade/Points
1	---/18
2	---/18
3	---/18
4	---/18
5	---/18
6	---/20
Total	--/110

1. (a) For the polar curve, $r = 1 - \cos\theta,$ find (9 Points)

   (i) singular point(s)
   (ii) the highest and lowest points.

(i)

(ii)

(b) Calculate area of the region that lies inside $r = 2 - 2\cos\theta$ and outside $r = 1$.

(9 Points)

2. (a) Suppose two lines have parametric equations:

$$l: \quad x = 2t + 1, \quad y = -t + 3, \quad z = 5t$$
$$m: \ x = 3 + 4t, \quad y = 2 - t, \quad z = 2t$$

(i) Find distance from $A(3,1,-1)$ to l.

(ii) Do the lines l and m intersect? If yes, find their point of intersection.

(9 Points)

(i)

(ii)

(b) The graph of $y = 4x^2$ is revolved about the $y-$axis. Find the equation of resulting surface S. Identify the surface S and give a rough sketch of this surface.

(5 Points)

(c) Sketch the level surface of:

$$f(x, y, z) = z - (x+2)^2 - (y-3)^2 + 16 \qquad \text{for } k = 7. \qquad \text{(4 Points)}$$

3. (a) The function $f(x,y) = x^2 y$ has a local linear approximation $L(x,y) = 4y - 4x + 8$ at a point $P_0(x_0, y_0)$. Find the point P_0.

(6 Points)

(b) Let f be a differentiable function of one variable. Assume that $z = f(x^2 + y^2)$.

Use the chain rule to calculate $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y}$. (6 Points)

(c) Let $f(x,y)=\tan^{-1}\left(y/x\right)$.

Find the degree n of the homogeneous function $f(x,y)$. Verify the formula
$xf_x(x,y)+yf_y(x,y)=nf(x,y)$. (6 Points)

4. (a) Find a point on the surface $z = 6 - 4x^2 - 3y^2$ at which the tangent plane is perpendicular to the line (5 Points)

$$x = 2 - 5t, y = 7 + t, z = 3 - 2t.$$

(b) Given that $\nabla f(x_0, y_0) = \vec{i} - 2\vec{j}$ and $D_{\vec{u}}(x_0, y_0) = -2.$ Find $\vec{u}.$

(4 Points)

(c) Examine the function $f(x,y) = x^3 + y^3 - 6xy$ for the local extrema and saddle points. (9 Points)

5. (a) Use the Lagrange multipliers to find the points on the sphere $x^2 + y^2 + z^2 = 9$ that are closest to and farthest from the point (2, 3, 4). (9 Points)

(b) Use an iterated double integral to find volume of the solid that is bounded by the parabolic cylinders (9 Points)

$z = x^2$, $y = x^2$ and $y = 8 - x^2$

6. (a) Convert $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)dz\ dx\ dy$ to cylindrical coordinates and evaluate the resulting repeated integral. (10 Points)

(b) Use the spherical coordinates to find the volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 4$ and the $xy-$plane. (10 Points)