

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester I, 2005-2006(051)
MATH 202
Major Exam 2
Date: December 6, 2005
Time: 1 hour and 30 minutes

Student Name: _____

Student ID: _____

Section: _____

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. Write down the substitutions required for solving the following equations. Show all the work.

(a) $x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$.

(b) $x \frac{dy}{dx} - (1 + x)y = xy^3$.

(c) $\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$.

2. A tank contains 200 liters of pure water. Brine containing 1 gram of salt per liter is pumped into the tank at a rate of 4L/min; the well mixed solution is then pumped out at the same rate. Find the number $A(t)$ of grams of salt in the tank at time t .

3. (a) Define the linear independence of functions.
- (b) Show that the functions $1, x^2, \cos x, \sin x$ are linearly independent.
- (c) Given that $y'' + y = x$ has a particular solution $y_{p_1} = x$, $y'' + y = e^x$ has a particular solution $y_{p_2} = \frac{1}{2}e^x$. Write down a particular solution of $y'' + y = x + 2e^x$.

4. (a) Find m so that e^{mx} is a solution of $xy'' - (x + 1)y' + y = 0$.
(b) Find a second solution by reduction of order.

5. (a) One solution of a second-order homogeneous linear differential equation is $y_1 = e^{-2x} \sin 4x$. Find the differential equation.
- (b) Find the annihilator of lowest order of $x + x^2 + e^{2x} + \sin 2x$.

6. Find the form of a particular solution of $y'' + 2y' + y = x^2 e^{-x}$.

7. Solve $y'' + 3y' + 2y = \frac{1}{1 + e^x}$.

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester I, 2005-2006(051)
MATH 202
Major Exam 2
Date: December 6, 2005
Time: 2 hours 30 minutes

Student Name: _____

Student ID: _____

Section: _____

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. Write down the substitutions required for solving the following equations. Show all the work.

(a) $x \frac{dy}{dx} + y = \sqrt{x^2 + y^2}.$

(b) $x \frac{dy}{dx} + (1 + x)y = xy^3.$

(c) $\frac{dy}{dx} = 2 + \sqrt{y + 2x + 3}.$

2. A tank contains 200 liters of pure water. Brine containing 1 gram of salt per liter is pumped into the tank at a rate of 3L/min; the well mixed solution is then pumped out at the same rate. Find the number $A(t)$ of grams of salt in the tank at time t .

3. (a) Define the linear independence of functions.
- (b) Show that the functions $1, x^2, \cos x, \sin x$ are linearly independent.
- (c) Given that $y'' + y = x$ has a particular solution $y_{p_1} = x$, $y'' + y = e^x$ has a particular solution $y_{p_2} = \frac{1}{2}e^x$. Write down a particular solution of $y'' + y = x + 4e^x$.

4. (a) Find m so that e^{mx} is a solution of $xy'' + (x + 1)y' + y = 0$.
(b) Find a second solution by reduction of order.

5. (a) One solution of a second-order homogeneous linear differential equation is $y_1 = e^{-4x} \sin 2x$. Find the differential equation.
- (b) Find the annihilator of lowest order of $x + x^2 + e^{3x} + \sin 2x$.

6. Find the form of a particular solution of $y'' - 2y' + y = x^2 e^{-x}$.

7. Solve $y'' + 3y' + 2y = \frac{1}{1 + e^x}$.