

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester I, 2005-2006(051)
MATH 202
Final Exam
Time: 3 hours

Student Name: _____

Student ID: _____

Section: _____

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. (a) Define ordinary and singular points of the differential equation

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0.$$

- (b) Find singular points of

$$x(x - 1)y'' + (\sin x)y' + y = 0.$$

- (c) The equation in (b) has a power series expansion $\sum_{n=0}^{\infty} a_n(x - 2)^n$. If R is the radius of convergence of this series, then $R \geq$ _____ . (15 points)

2. A function y satisfies $y''(x) + (\sec x)y' + (\tan x)y = 0$. If y has a power series expansion $y = \sum_{n=0}^{\infty} a_n x^n$, $y(0) = 1$, $y'(0) = -1$, find a_0, a_1, a_2, a_3 , using the given differential equation. (10 points)

3. (a) Define what it means for x_0 to be a regular singular point of a differential equation

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0.$$

- (b) Verify that the indicial equation of $xy'' + 2y' - xy = 0$ is $r(r + 1) = 0$.
- (c) Give a power series solution of the equation in (b) for the larger root of the indicial equation. (30 points)

4. (a) Verify that $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ has only one eigenvalue λ .
- (b) For this eigenvalue, find the eigenvectors.
- (c) Find all vectors \vec{v} so that $(A - \lambda I)^2 \vec{v} = 0$. (λ as in part (a)). Find a fundamental set of such vectors and write down the corresponding fundamental solution of the equation $dX/dt = AX$: here A is as in part (a). (20 points)

5. Find the general solution of $dX/dt = \begin{pmatrix} 4 & -5 \\ 5 & -4 \end{pmatrix} X$. (10 points)

6. The equation $y''(x) + a(x)y'(x) + b(x)y = 0$ has fundamental solutions $y_1 = \cos x$, $y_2 = \sin x$. Using variation of parameters, find a particular solution of the inhomogeneous equation $y''(x) + a(x)y'(x) + b(x)y = x$. (10 points)

7. (a) When is a differential equation $P(x, y)dx + Q(x, y)dy = 0$ an exact equation?
- (b) Solve the differential equation $(x + \sin x + \sin y)dx + \cos y dy = 0$ by finding an integrating factor of the form $\mu = \mu(x)$. (15 points)

8. (a) Define linear independence of functions f_1, f_2, f_3 .
- (b) Show that the functions $f_1 = e^x$, $f_2 = xe^x$, $f_3 = x^2e^x$ are linearly independent.
(10 points)

9. Find the general solution of the differential equation $y'' + y = e^x$. (10 points)

10. Solve the initial value problems:

(a) $\frac{dy}{dx} = \cos(x + y), \quad y(0) = \frac{\pi}{4}.$

(b) $\frac{dy}{dx} = \frac{1 - 2x - y}{2 + 2x + y}, \quad y(0) = 0.$

If your solution is implicit, **do not** make it explicit.

(20 points)