MATH 302 -01+03 (Term 051)

Major Exam II

Wednesday 7 December, 2005

Time Allowed: 75 minutes

Instructor: Dr. A. Boucherif

Name:..........................................................ID #............................

Section#:............Serial #:............

• Write clearly.
• Show all your steps.
• No credit will be given to wrong steps.
• Do not do messy work
• Calculators are not allowed in this exam.

Mark: 20
Q1. Compute the maximum and the minimum rate of change of \( \varphi(x, y, z) = -1 / \|xi + yj + zk\| \) at the point \( P = (-2, 1, 1) \).

Q2. Let \( A = ai + bj + ck \) be a constant vector and \( R = xi + yj + zk \). Prove that (a) \( \nabla (A.R) = A \), (b) \( \nabla \cdot (R - A) = 3 \), (c) \( \nabla \times (R - A) = 0 \).

Q3. Let \( F = x^2yi - xy^2j \) and let \( C \) be the boundary of the region \( x^2 + y^2 \leq 4, x \geq 0, y \geq 0 \). Evaluate \( \int_C F.dR \) in two different ways (directly and using Green’s theorem).

Q4. Let \( F = \frac{y}{x}i + (\ln x)j \) and let \( C \) be any path connecting \( P_0 = (1, 1) \) to \( P_1 = (2, 2) \). Under what condition on the path \( C \) we can evaluate \( \int_C F.dR \). Find the value of the line integral.

Q5. Let \( \sum \) be the part of the paraboloid \( z = 4 - x^2 - y^2 \) lying above the \( xy \)-plane. Evaluate \( \int_{\sum} x^2 d\sigma \).

Q6. Let \( F = -8y^2i - (16xy + 4z)j - 4yk \). Compute \( \nabla \times F \).
(a) What can you conclude about the vector field \( F \).
(b) Find \( \phi \) so that \( \nabla \phi = F \), and
(c) evaluate the line integral of \( F \) along any path connecting \( P_0 = (-2, 1, 1) \) to \( P_1 = (1, 3, 2) \).