(1) Consider the graph $G = K_{2,3,2}$. Answer each of the following. If your answer is no, explain why; and if yes, support it by construction or calculation.

(a) Is $G$ Eulerian graph?
(b) Is $G$ Hamiltonian graph?
(c) Is $G$ nonseparable?
(d) What is the number of edges in a maximum matching of $G$?
(e) Does $G$ have a perfect matching?
Let $G$ be a nontrivial labeled graph with vertices $\{v_1, v_2, ..., v_n\}$. Let $A$ and $D$ respectively denote the adjacency and the degree matrices of $G$ and let $A^n = (a^n_{ij})$. What is the graph theoretical meaning of:

(a) $a^n_{ij}$.

(b) $\frac{1}{2 \times 3} \sum_{i=1}^{n} a^3_{ii}$.

(c) The value of any cofactors of the matrix $D - A$.

(3) Give an equivalent condition for a matching $M$ to be maximum matching. Define any terminology you use.
(4) Either prove or give a counterexample for each of the following statements:

(a) Every cubic graph contains a 1-factor.

(b) If a graph $G$ has exactly two vertices of odd degree, then they are connected by a path.

(c) There is a graph $G$ of order 4 with $\chi(G) = 2$ and $\chi(\overline{G}) = 1$.

(d) If $v$ is a cut-vertex of a connected graph $G$, then $v$ is not a cut-vertex of $\overline{G}$. 
(e) If $G_1$ and $G_2$ are regular graphs, then $G_1 + G_2$ is regular.

(f) If $G$ is a self-complementary graph with $n$ vertices, then $n = 4k$ for some integer $k$.

(g) Every tree has a perfect matching.

(h) There is a nontrivial graph $G$, all of whose vertices have different degrees.
(5) Prove each of the following:
(a) If $G$ is connected $(n, m)$–graph with $m = n - 1$, then $G$ is a tree.
(b) Every transitive tournament is acyclic.
(c) The Petersen graph is not 1–factorable.
(d) The automorphism group of $P_5$ is $Z_2$. 

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