King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2005-2006 (052)
MATH 101 — Final Exam

Code 001

Name: ID: Section:

Q1. $\lim\limits_{x\mapsto -\infty} \frac{x^3-x^2+1}{2-x-x^2} =$

(a) 0

(b) 1

(c) -1

(d) $-\infty$

(e) $+\infty$

Q2. The area of the triangle formed by the tangent line to the curve $xy = 1$ at $x = -3$ and the coordinate axes is equal to

(a) $5/3$

(b) 2

(c) $-1/9$

(d) $7/3$

(e) 3

Q3. The derivative $\frac{dy}{dx}$ of $y = \cosh^{-1}\sqrt{x^2+1}$ is equal to

(a) $\frac{1}{\sqrt{x^2+1}}$

(b) $\frac{1}{x\sqrt{x^2+1}}$

(c) $\frac{x}{|x|\sqrt{x^2+1}}$

(d) $\frac{-1}{\sqrt{x^2+1}}$

(e) $\frac{1}{\sqrt{1-x^2}}$

Q4. The linear approximation of the function $f(x) = (1+x)^{1/5}$ at 0 is

(a) $\frac{x}{5} + 1$

(b) $\frac{x}{5} - 1$

(c) $-\frac{x}{5} - 1$

(d) $-\frac{x}{5} + 1$

(e) $x + \frac{1}{5}$

Q5. If the derivative of a function f is differentiable for all x, then

$$\lim_{x\to 1} \frac{\lim_{h\to 0} \frac{f(1+h)-f(1)}{h/2} - 2f'(x)}{\frac{x}{2} - \frac{1}{2}} =$$

(a) $-f'(1)$

(b) $2f''(\frac{x}{2})$

(c) $\frac{f''(1)}{4}$

(d) $-\frac{1}{2}f'(\frac{x}{2})$

(e) $-4f''(1)$

Q6. The set of x-coordinates of the points at which the function $f(x) = \frac{x-1}{x^4-2x^3-x+2}$ has removable discontinuity is:

(a) $\{1\}$

(b) $\{2\}$

(c) $\{1, 2\}$

(d) $\{-2, -1\}$

(e) $\{-2, 1\}$

Q7. Regarding y as the dependent variable and x as the independent variable in the equation $x^y = y^x$, the derivative $\frac{dy}{dx}$ is equal to

(a) $\frac{yx^{y-1}}{xy^{x-1}}$

(b) $\frac{xy^{x-1}}{yx^{y-1}}$

(c) $\frac{x \ln y - y}{y \ln x - x}$

(d) $\frac{y^2 - xy \ln y}{x^2 - xy \ln x}$

(e) $\frac{\ln y - \frac{y}{x}}{\ln x}$

Q8. The function $f(x) = -\frac{1}{4}x^4 + 2x^2 + 3$ has exactly

(a) one local maximum and one local minimum

(b) two local minimums and one local maximum

(c) one local minimum and two local maximums

(d) one local minimum and no local maximums

(e) one local maximum and no local minimum

Q9. Using Newton's method with initial approximation $x_1 := -1$, the second approximation x_2 of the root of the equation $5x^3 - 3x^2 + x + 2 = 0$ is:

(a) $-\frac{15}{22}$

(b) $-\frac{1}{2}$

(c) $-\frac{10}{3}$

(d) $-\frac{1}{4}$

(e) $-\frac{1}{6}$

Q10. $\lim_{x \mapsto 1^-} \frac{1-x^2}{x^3-1} =$

(a) $\frac{1}{3}$

(b) $-\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{-2}{3}$

(e) ∞

Q11. If the normal line to the curve $y = f(x)$ at the point $(-1, 3)$ passes through the point $(0, 5)$, then $f^{'}(-1) =$

(a) -2

(b) $3/2$

(c) $1/2$

(d) $-1/2$

(e) -1

Q12. The largest possible δ such that

$$|x-1| < \delta \Rightarrow \left|x^2 - 1\right| < \frac{1}{2}$$

is:

(a) $\sqrt{\frac{3}{2}} - 1$

(b) $1 - \frac{1}{\sqrt{2}}$

(c) $\sqrt{\frac{3}{2}}$

(d) $\frac{1}{\sqrt{2}}$

(e) 1

Q13. If $f(x) = \frac{\sin x \sec x}{1+x \tan x}$, then $f^{'}(0) =$

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

Q14. $D^{1000}(xe^{-x})$ is equal to

(a) $(x - 1001)e^{-x}$

(b) $(1000 - x)e^{-x}$

(c) $(999 - x)e^{-x}$

(d) $(x - 1000)e^{-x}$

(e) $(1001 - x)e^{-x}$

Q15. The equation $x^{201} + 804x = 9 - \frac{402}{101}x^{101}$ has

(a) exactly one real solution

(b) no real solutions

(c) exactly 201 real solutions

(d) three real solutions

(e) one real solution in $(-\infty, 0)$ and another one in $(0, +\infty)$

Q16. A point of the curve of $y = \ln(\ln x)$ at which the tangent has slope $\frac{1}{e}$ is

(a) $(\frac{1}{e}, 0)$

(b) $(e^e, 1)$

(c) $(e, 0)$

(d) Does not have

(e) $(e^2, \ln 2)$

Q17. The function $f(x) = \frac{x^2+5}{x+2}$

(a) changes its concavity from concave down to concave up and has no inflection points

(b) changes its concavity from concave down to concave up and has an inflection point at -2

(c) changes its concavity from concave up to concave down and has no inflection points

(d) changes its concavity from concave up to concave down and has an inflection point at -2

(e) f is always concave up

Q18. A sketch of the curve of $f(x) = x^2 e^{-x^2}$ would be

Q19. Which one of the following curves corresponds to the function

$$f(x) = \frac{x^5}{(x^2-1)^2}.$$

Q20. The cost of a printer is 625 Riyals, and its value is depreciating (i.e. decreasing) with time according to the formula

$$\frac{dV}{dt} = -800(2t+1)^{-2},$$

where V Riyals is its value t years after its purchase. What is the value of the printer three years after its purchase (rounded to two decimals)?

(a) 280.33 Riyals

(b) 292.04 Riyals

(c) 272.74 Riyals

(d) 282.14 Riyals

(e) 290.04 Riyals

Q21. The set of vertical asymptotes for the graph of

$$f(x) = \frac{x^4 - 1}{x^3 - x}$$

is:

(a) $\{x = -1,\ x = 0,\ x = 1\}$

(b) $\{x = -1,\ x = 0\}$

(c) $\{x = 0,\ x = 1\}$

(d) $\{x = -1,\ x = 1\}$

(e) $\{x = 0\}$

Q22. The derivative $\frac{dy}{dx}$ for $y = \sqrt{x + \coth\sqrt{1+x^2}}$ is:

(a) $\frac{\sqrt{1+x^2}+csch^2(\sqrt{1+x^2})}{2\sqrt{1+x^2}\sqrt{x+\coth(\sqrt{1+x^2})}}$

(b) $\frac{1-csch^2(\sqrt{1+x^2})}{2\sqrt{x+\coth(\sqrt{1+x^2})}}$

(c) $\frac{-x\cosh^2(\sqrt{1+x^2})}{2\sqrt{x+\coth(\sqrt{1+x^2})}}$

(d) $\frac{1}{2\sqrt{x+\coth(\sqrt{1+x^2})}}$

(e) $\frac{\sqrt{1+x^2}-x\ csch^2(\sqrt{1+x^2})}{2\sqrt{1+x^2}\sqrt{x+\coth(\sqrt{1+x^2})}}$

Q23. The equation of the line through the point $(2,3)$ that forms with the positive x-axis and the positive y-axis a triangle of non-zero least area is:

(a) $y = -\frac{2}{3}x + \frac{13}{3}$

(b) $y = -3x + 9$

(c) $y = -\frac{3}{2}x + 6$

(d) $y = -x + 5$

(e) $y = 2x - 1$

Q24. Which one of the following statements is always true?

(a) If f is continuous on $[-1,1]$ and $f(-1) = 4$ and $f(1) = 3$, then there exists a number r such that $|r| < 1$ and $f(r) = \pi$

(b) If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1

(c) If f is continuous at a, then f is differentiable at a

(d) If $f(x) > 1$ for all x and $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x) > 1$

(e) If $\lim_{x\to 6} f(x)g(x)$ exists, then it is equal to $f(6)g(6)$

Q25. Water is leaking out (escaping) of an inverted conical (cone shape) tank at a rate of $10^4 cm^3/\min$. At the same time water is being pumped into the tank at a constant rate. The height of the tank is $H = 8m$ and its diameter at the top is $D = 6m$. If the water level is rising at the rate of $\frac{64}{81\pi} cm/\min$ when the height h of the water is $h = 3m$, the rate at which the water is being pumped into the tank is

(a) $20000\ cm^3/\min$

(b) $10000\ cm^3/\min$

(c) $15000\ cm^3/\min$

(d) $10000.\ \frac{64}{81\pi} cm^3/\min$

(e) $\left(10000 + \frac{64000}{81\pi}\right) cm^3/\min$