

Serial No:

Student No.:

Name:

- 1. SHOW ALL WORK. NO CREDITS FOR ANSWERS NOT SUPPORTED BY WORK.**
2. CALCULATORS ARE NOT ALLOWED.

Problem 1 (25 Points):

A. If the limit exists find it. If the limit does not exist, say so; use ∞ and $-\infty$ when appropriate.

(a) $\lim_{x \rightarrow 1^-} \frac{x-1}{x^2-2x+1}$

(b) $\lim_{x \rightarrow 0^+} \frac{x^2}{\sqrt{x^2+4}-2}$

(c) $\lim_{x \rightarrow -\infty} \frac{5x^2 - \sqrt{x} - 9}{120x - 3x^2}$

B. Let $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$. For each of the following, find the limit. If the limit does not exist, say so; use ∞ and $-\infty$ when appropriate.

(i) $\lim_{x \rightarrow 0^-} f(x)$

(ii) $\lim_{x \rightarrow 0^+} f(x)$

(iii) $\lim_{x \rightarrow 0} f(x)$

(iv) $\lim_{x \rightarrow \infty} f(x)$

Problem 2 (25 Points)

(a) Use the definition of the derivative to find $f'(2)$ for the function $f(x) = 5x - 3$.

(b) Find the equation of the line tangent to the graph of $y = \left(\frac{2x^2}{1+x}\right)^4$ at the point (1, 1).

(c) Find all points on the graph of $y = x^3 - 3x^2 + 1$ where the slope is 0.

Problem 3 (25 Points)

(a) Find all value(s) of x for which the following function is *discontinuous*.

$$f(x) = \begin{cases} \frac{1}{x^2 - 1} & \text{if } x < 0 \\ -1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases} .$$

(b) The average cost \bar{c} of producing q units of a product is given by $\bar{c} = \frac{4q}{q+2} + \frac{10000}{q}$. Find the marginal cost function.

(c) For the consumption function $C = 10 + \frac{5}{8}I - \frac{\sqrt{I}}{2}$,

a. find the marginal propensity to consume when $I = 16$;

b. find the marginal propensity to save when $I = 16$.

Problem 4 (25 Points):

(a) Find the derivative of $y = (x^2 + 9)\sqrt{x^3 + 4}$ at $x = 0$.

(b) If $y = u^3 + u^2 + 7u - 2$ and $u = 5x - 6$, find $\frac{dy}{dx}$ when $x = 1$.

(c) Find the rate of change in the area of a circle with respect to the radius when the radius is 2 inches. Also find the relative rate of change in the area. ($A = \pi r^2$)