

King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences
Semester (052)
MATH 202.04, MATH 202.06
Major Exam 2

Name: _____ ID: _____

Section: _____

Question #	Points obtained	Maximum points
1		20
2		15
3		15
4		20
5		15
6		15
Total		100

Calculators are not allowed.

Time allowed: 2 hours

Instructor: Prof. Hassan Azad
Date: April 19, 2006

Q.1.

- a) Define linear independence of functions f_1, f_2, f_3, f_4, f_5
- b) Show that the functions x, x^2, x^3, x^4, e^x are linearly independent.
- c) Functions u_1, u_2, u_3 satisfy the equations

$$u_1'' + p(x)u_1' + q(x)u_1 = 2e^x + 1$$

$$u_2'' + p(x)u_2' + q(x)u_2 = 4$$

$$u_3'' + p(x)u_3' + q(x)u_3 = 3x$$

Write down a particular solution of $y'' + p(x)y' + q(x)y = e^x + x + 1$, in terms of functions u_1, u_2, u_3 .

Q.2. Find the general solution of the differential equation

$$y'' - 2y' + y = e^x \ln x \quad (x > 0)$$

[Use variation of parameters]

Q.3. Find the form of a particular solution of $y'' + 4y = 2x^2 + 5\sin 2x + e^{3x}$.

15

Q.4.

- a) Find annihilators of lowest order of $x + x^2 + x^3 + \cos x$.
- b) Find annihilator of $\cos^2 x$.
- c) Find annihilator of $\cos^3 x$.

6+6+8

5

Q.5.

- a) The function y_1 is a non-zero solution of $a(x)y'' + b(x)y' + c(x)y = 0$ and $y_2 = uy_1$ is a solution obtained by reduction of order. Explain why y_1 and y_2 are linearly independent.
- b) One solution of the equation $y'' - \left(2 + \frac{n-1}{x}\right)y' + \left(1 + \frac{n-1}{x}\right)y = 0$ is $y_1 = e^x$ (here n is a positive integer). Find the second solution by reduction of order.

Q.6. A particle moves along a line so that its velocity at time t is proportional to the square of its position $x(t)$ at time t .

- a) If $x(0) = -1$, find $x(t)$.
b) Find limit of $x(t)$ as $t \rightarrow \infty$.

12+3
