1) (a) Show that \( u(x, y) = x^3 - 3x y^2 \) is harmonic function and find its harmonic conjugate.

(b) Determine the domain of analyticity of \( f(z) = \text{Log}(4 + i - z) \). Compute \( f'(z) \).

2) Find residues at isolated singularities

(a) \( f(z) = \frac{1}{z^2 e^{-z}} \)

(b) \( f(z) = \frac{\sin z}{z^4(z - 1)} \)

3) Evaluate the following using the complex integration

\[
PV \int_{-\infty}^{\infty} \frac{x \cos x}{x^2 + 2x + 2} \, dx
\]

4) Evaluate \( PV \int_{-\infty}^{\infty} \frac{e^{\frac{1}{1+e^x}}}{x} \, dx \)

5) Show that \( \int_{0}^{\infty} \frac{\sqrt{x}}{x^2 + 1} \, dx = \frac{\pi}{\sqrt{2}} \)

6) Find value of

\[
PV \int_{0}^{\infty} \frac{\text{Log}x}{x^2 + 1} \, dx
\]