1. Given the first order linear equation \( y'(t) = p(t)y(t) + q(t) \), where \( y \) is an unknown scalar function, \( p(t) \), \( q(t) \) are continuous on the interval \([a, b]\), show that the solution satisfying \( y(t_0) = y_0 \) is given by

\[
y(t) = \exp\left[\int_{t_0}^{t} p(s)ds\right]\{y_0 + \int_{t_0}^{t} q(s) \exp\left[-\int_{t_0}^{s} p(u)du\right]ds\}
\]

Application. Use this formula to find the solution of \( y'(t) = -t^{-1}y(t) - t^{-2}, \ y(1) = 1 \).

2. Suppose, now, that \( p \) and \( q \) are periodic of period \( T \). Show that the above equation has a periodic solution if and only if \( \exp \int_{0}^{T} p(s)ds \neq 1 \). Find the periodic solution of \( y'(t) = ty(t) + \sin t \).

3. Consider the system \( x' = -x - y/\ln(x^2 + y^2)^{1/2}, \ y' = -y + x/\ln(x^2 + y^2)^{1/2} \).
   (i) Show that \((0, 0)\) is the only critical point.
   (ii) Show that \((0, 0)\) is a spiral point, whereas it is a proper node for the corresponding linear system.
Given the system

\[ x' = -y + x (x^2 + y^2) \sin \frac{\pi}{\sqrt{x^2 + y^2}} \]
\[ y' = x + y (x^2 + y^2) \sin \frac{\pi}{\sqrt{x^2 + y^2}} \]

1. Show that \((0, 0)\) is the only simple critical point.
2. Use polar form to show that the family of circles \(C_n : x^2 + y^2 = \frac{1}{n^2}, n = 1, 2, 3, \ldots\) are orbits and it is the only family of closed orbits.
   (To prove the last part show that for \(\frac{1}{n+1} < r < \frac{1}{n}\) trajectories spiral away from or toward the origin, and for \(r > 1\) trajectories become unbounded).

**Solutions.**