King Fahd Univ. of Petroleum and Minerals  
Faculty of Sciences  
Department of Mathematical Sciences

FINAL EXAM  
(MATH. 260-053 Sections 2 and 3)

Name:

ID:

Important instructions:
- Use an HB pencil or a pen (do not use red color)
- Solve the problems completely
- Write down your answers in a clear manner
- Justify all your steps
- Use the back of the page (verso) only for scratching
Prob. 1
Find a general solution for

\[(t^2 + 1)dx = t \cot xdt\]
Prob. 2
Solve
\[ \frac{dz}{dy} = 1 + y + z + yz, \quad z(0) = 0 \]
Prob. 3
Solve

\[(1 + 2xy) \frac{dy}{dx} = 1 + y^2\]
Prob. 4

Solve

\[(e^u \sin v + \tan v)du + (e^u \cos v + \sec^2 v)dv = 0\]
Prob. 5

Is

\[ E = \{(x_1, x_2, x_3, x_4) : x_1 = 3x_3 \text{ and } x_2 = 4x_4\} \]

subspace of \( \mathbb{R}^4 \)? Justify your answer!
Prob. 6
The vectors \( \{v_1, v_2, v_3\} \) are known to be linearly independent. Prove that

\[
\begin{align*}
    u_1 &= v_1 \\
    u_2 &= v_1 + 2v_2 \\
    u_3 &= v_1 + 2v_2 + 3v_3
\end{align*}
\]

are also linearly independent.
Prob. 7

Set up the appropriate form of a particular solution $y_p$, but do not determine the values of the coefficients

a) $y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$

b) $y^{(4)} + 5y'' + 4y = \sin x + 2 \cos 2x$
Prob. 8
Use the method of variation of parameters to solve

\[ y'' + y = \csc^2 x. \]
Prob. 9

Is

\[ A = \begin{pmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ -4 & 4 & 1 \end{pmatrix} \]

diagonalizable? If it is, find \( P \) and \( D \).
Prob. 10
Solve by the eigenvalue method

\[
\begin{align*}
x'_1 &= 5x_1 + 5x_2 + 2x_3 \\
x'_2 &= -6x_1 - 6x_2 - 5x_3 \\
x'_3 &= 6x_1 + 6x_2 + 5x_3.
\end{align*}
\]