KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Major Exam 1

Math 302
(Engineering Mathematics)

Time Allowed: 1 Hour

Student Name:_______________    Id. No. _______________

Section:_________________

Note

No programmable calculators and mobile phones allowed in the examination hall. For all questions show calculations in support of your answers.

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Instructor Name
Ashfaque H. Bokhari
Q 1  Solve the system

\[
\begin{align*}
2x_1 + 2x_2 &= 2 \\
2x_1 + 2x_3 &= b \\
-x_1 - x_2 + bx_3 &= 2
\end{align*}
\]

and give answer to the following questions:

(a): Find condition on “b” under which the system is consistent. (2 points)
(b): Write solution in vector form. (1 point)
Q 2  Suppose $S$ is a subspace of $\mathbb{R}^4$ whose elements satisfy the condition

$$x_1 + ax_2 - bx_3 - cx_4 = 0$$

(a):  Give dimension and vectors spanning the subspace “S”.  
(b):  Using determinant method show that vectors are linearly independent.
Q3. Find Eigenvalues and Eigenvector of the matrix given by
\[
\begin{pmatrix}
6 & 8 & 0 \\
-1 & 2 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
(5 points)
and if possible diagonalize it.
Q4. Find unit tangent vector (parameterized by arc length) to the curve defined by the position vector
\[ r(t) = (6 \sin 2t, 6 \cos 2t, 5t) \quad 0 \leq t \leq 3\pi \] (5 points)
Q5. The matrix $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ has Eigenvalues (3, 1) and corresponding Eigenvectors 
$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Using Eigenvector matrix compute $A^3$. (3 points)