King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester I, 2006-2007(061)
MATH 201 Secs. 4 & 7
Major Exam II

Student Name: ______________________________

Student ID: ______________________________

Section: ______________________________

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. (a) Let L_1, L_2 be lines whose parametric equations are

$$\begin{aligned} L_1: &\quad x = 4t, y = 1 - 2t, z = 2 + 2t \\ L_2: &\quad x = 1 + t, y = 1 - t, z = -1 + 4t \end{aligned}$$

Show that these lines intersect.

(b) Find parametric equations of the line perpendicular to L_1, L_2 which passes through their point of intersection.

2. Find equation of the plane through the point $(-1, 4, 2)$ that contains the line of intersection of the planes $4x - y + z - 2 = 0$ and $2x + y - 2z - 3 = 0$.

3. Show that the set of points equidistant from the planes $x + y = 0$ and $y + z = 0$ is a union of two perpendicular planes.

4. identify the surfaces

(a) $4z^2 = x^2 + 4y^2$.

(b) $9x^2 + y^2 + 4z^2 - 18x + 2y + 16z = 0$.

(c) Draw a rough sketch of the surfaces in parts (a) and (b).

5. Express the following equations in rectangular coordinates and identify them:

(a) $\rho \sin \phi = 1$.

(b) $\rho - 2 \sin \phi \cos \theta = 0$.

6. Evaluate the limit, if it exists, by converting to spherical coordinates

(a) $$\lim_{(x,y,z)\to(0,0,0)} \frac{e^{\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}}.$$

(b) $$\lim_{(x,y,z)\to(0,0,0)} \tan^{-1}\left[\frac{1}{x^2+y^2+z^2}\right].$$

7. (a) Define differentiability of a function of 2 variables at a point.

   (b) Verify, using the definition, that $f(x, y) = x^2 + y^2$ is differentiable at any point (x, y).

8. (a) Use a chain rule to find the values $\left.\frac{\partial z}{\partial r}\right|_{r=2,\ \theta=\pi/6}$ and $\left.\frac{\partial z}{\partial \theta}\right|_{r=2,\theta=\pi/6}$ if $z = xye^{x/y}, x = r\cos\theta, y = r\sin\theta$.

(b) Let f be a differentiable function of one variable and let $w = f(u)$, where $u = x + 2y + 3z$. Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 6\frac{dw}{du}$.