King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester I, 2004-2005(041)
MATH 201
Final Exam
Time: 3 hours

Student Name: _____

Student Number: _____

Section #: _____

Serial #: _____

Note:

- FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.
- Use of any calculator not allowed.

1. (a) Find equation of the surface obtained by rotating the curve (in the xy-plane) $x^2 - y^2 = 1$ about the x-axis. Draw a rough sketch of the surface. (5 points)

   (b) Draw the plane $2x + y + z = 5$. (5 points)

   (c) Draw the plane $x + y = 2$. (2 points)

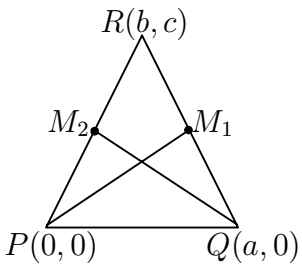
   (d) Draw the surface $x = y^2$. (3 points)

2. Fill in the following table:

Equation of Surface in 3 dimensions	Mathemaitcal Name	Sketch of surface
$z = y^2 + 2$		
$4x^2 - y^2 = 4z^2$		
$z - 1 = x^2 + y^2$		
$x^2 + y^2 - z^2 = 1$		

(10 points)

3. A triangle has vertices $P(0,0)$, $Q(a,0)$ and $R(b,c)$. Find the point of intersection of the lines PM_1 and QM_2, where M_1 is the mid-point of RQ and M_2 is the mid-point of PR. (10 points)



4. (a) Find the parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $x^2 + 4y^2 + z^2 = 9$ at the point $(1, -1, 2)$. (10 points)

   (b) Let $w = f(x - y, y - z, x - z)$. Show that $\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z} = 0$. (5 points)

5. Locate all relative maxima, relative minima and saddle points, if any, of

$$f(x, y) = x^2 + y - e^y.$$

(10 points)

6. Use polar coordinates to find the volume below the surface $z = \sqrt{x^2 + y^2}$, inside the cylinder $x^2 + y^2 = 2y$ and above the plane $z = 0$. (10 points)

7. (a) Describe the solid below the paraboloid $z = x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 4$ by inequalities. (5 points)

   (b) Find the volume of the solid in part (a). (10 points)

8. Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 1$. (10 points)

9. Find the volume of the solid outside the sphere $x^2+y^2+z^2=1$ but inside the sphere $x^2+y^2+(z-1)^2=1$ (Use spherical coordinates: the volume element in spherical coordinates is $dv=\rho^2\sin\phi\ d\rho\ d\phi\ d\theta$. (15 points)