

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 201 (Calculus III)(061)
Major Exam II (Sec. # 3, 6, 13)
December 13, 2006

Time: 90 Minutes

Marks: /60

Name: _____

Serial #: _____

ID #: _____

Section #: _____

- This exam consists of 9 pages.
- Show complete and neat work for full credit.
- Use of graphic calculators is not allowed in this exam.

1. Check whether the lines are parallel or skew.

$$L_1 : \quad x = 3 + t, \quad y = 2 - 4t, \quad z = t$$

$$L_2 : \quad x = 4 - t, \quad y = 3 + t, \quad z = -2 + 3t$$

(5 points)

2. Find equation of a plane P that contains the point $P_0(4, -3, 0)$ and the line $x = 3t + 1$, $y = -2t$, $z = t - 3$. Also calculate distance between the point P_0 and the plane P . (10 points)

3. (a) Find equation of surface of revolution by revolving the graph of the equation $y = 4x^2$ about y -axis. Give a rough sketch of the surface. (5 points)

- (b) Describe and sketch curve of intersection between the paraboloids $z = x^2 + y^2$ and $z = 6 - x^2 - y^2$. (5 points)

4. Define $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $f(x, y) = \begin{cases} \frac{\sin(2x^2 + y^2)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases}$

(5 points)

(a) Is $f(x, y)$ continuous at $(0, 0)$? Give reasons.

(b) Is $f(x, y)$ differentiable at $(0, 0)$? Justify.

5. The function $f(x, y) = x^2y$ has a local linear approximation $L(x, y) = 4y - 4x + 8$ at a point $P_0(x_0, y_0)$. Find the point P_0 . (5 points)

6. (a) Let $z = \tan^{-1}\left(\frac{u}{v}\right)$ where $u(x, y) = 2x + y$ and $v(x, y) = 3x - y$. Find $\frac{\partial z}{\partial y}$.
(5 points)

- (b) Given that $\nabla f(x_0, y_0) = \vec{i} - 2\vec{j}$ and $D_{\vec{u}} f(x_0, y_0) = -2$. Find \vec{u} . (5 points)

7. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $x^2 + 4y^2 + z^2 = 4$ at the point $(1, -1, 2)$.
(5 points)

8. Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$.
(10 points)