Name: _______________________________  Serial #: __________

ID #: _______________________________  Section #: _________

• This exam consists of [9] pages.
• Show complete and neat work for full credit.
• Use of graphic calculators is not allowed in this exam.
1. Check whether the lines are parallel or skew.

\[ L_1 : \quad x = 3 + t, \; y = 2 - 4t, \; z = t \]
\[ L_2 : \quad x = 4 - t, \; y = 3 + t, \; z = -2 + 3t \]

(5 points)
2. Find equation of a plane $P$ that contains the point $P_0 (4, -3, 0)$ and the line

$x = 3t + 1, \ y = -2t, \ z = t - 3$. Also calculate distance between the point $P_0$ and the plane $P$. (10 points)
3. (a) Find equation of surface of revolution by revolving the graph of the equation

\[ y = 4x^2 \]

about y-axis. Give a rough sketch of the surface.

(b) Describe and sketch curve of intersection between the paraboloids

\[ z = x^2 + y^2 \]

and \( z = 6 - x^2 - y^2 \).
4. Define \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) by \( f(x, y) = \begin{cases} \frac{\sin(2x^2 + y^2)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases} \) (5 points)

(a) Is \( f(x, y) \) continuous at \((0, 0)\)? Give reasons.

(b) Is \( f(x, y) \) differentiable at \((0, 0)\)? Justify.
5. The function \( f(x, y) = x^2 y \) has a local linear approximation \( L(x, y) = 4y - 4x + 8 \) at a point \( P_0(x_0, y_0) \). Find the point \( P_0 \). (5 points)
6. (a) Let \( z = \tan^{-1}\left(\frac{u}{v}\right) \) where \( u(x, y) = 2x + y \) and \( v(x, y) = 3x - y \). Find \( \frac{\partial z}{\partial y} \).

(5 points)

(b) Given that \( \nabla f(x_0, y_0) = \vec{i} - 2\vec{j} \) and \( D_{\vec{u}} f(x_0, y_0) = -2 \). Find \( \vec{u} \).

(5 points)
7. Find parametric equations for the tangent line to the curve of intersection of the paraboloid \( z = x^2 + y^2 \) and the ellipsoid \( x^2 + 4y^2 + z^2 = 4 \) at the point \( (1, -1, 2) \).

(5 points)
8. Locate all relative extrema and saddle points of \( f(x, y) = 4xy - x^4 - y^4 \). (10 points)