King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math 201, Sections: 3, 6, 13
Final Examination
Semester I, 2006–2007(061)
January 28, 2007
Time: 07:00 p.m. to 09:30 p.m.

Name: ____________________ Section #: __________

ID #: ______________ Serial #: ____________________

Sec. #	Instructor	Location
03, 06, 13	Dr. Abdul Rahim Khan	Building 54 (Exhibition Center)

Instructions:

1. Do not use programmable calculators. Use of ordinary calculator is allowed.
2. Show all your work. Less credit will be given for answer not supported by proper work.
3. Clearly indicate the theorem or result you use.
4. This exam consists of 10 pages.
5. Do not forget to write your NAME, ID #, Section # and Serial # in the place provided above.

Grade: __________/110

1. Find the surface area generated by revolving the curve

$$C: \quad x = e^t \sin t,\ y = e^t \cos t, \quad 0 \le t \le \frac{\pi}{2}$$

about the y-axis. (12 points)

2. (a) Show that $\vec{u} \cdot \vec{v} = \frac{1}{4}\left[\|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2\right]$. (6 points)

(b) Find the equation of the plane through the points $P(1,0,-1)$ and $Q(2,1,0)$ that is parallel to the line of intersection of the planes $x+y+z=5$ and $3x-y=4$. (6 points)

3. Let $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$.

Check whether or not $f_{xy}(0,0) = f_{yx}(0,0)$. (12 points)

4. The directional derivative of $f(x, y, z)$ at the point $(-3, 1, 2)$ in the direction of $\vec{a} = 2\,\vec{\imath} - \vec{\jmath} - 2\,\vec{k}$ is -5. If $\|\nabla f(-3, 1, 2)\| = 5$, then find $\nabla f(-3, 1, 2)$. (12 points)

5. Use the Lagrange multiplier method to find the maximum and minimum values of the function $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 4$. (12 points)

6. Use double iterated integrals to find area of the region inside the cardioid $r = 2 + 2\cos\theta$ and to the right of the line $r\cos\theta = \dfrac{3}{2}$. (12 points)

7. (a) Change the triple integral to cylindrical coordinates (do not evaluate the resulting integral):

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)dz\,d\,xd\,y.$$

(6 points)

(b) Set up a triple integral in spherical coordinates to find volume of the solid that lies outside the cone $z^2 = x^2 + y^2$ and inside the sphere $x^2 + y^2 + z^2 = 1$.

(6 points)

8. Use spherical coordinates to evaluate:

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z^2(x^2+y^2+z^2)^{1/2}\,dz\,dy\,dx.$$

(12 points)

9. Use spherical coordinates to find volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$., the cylinder $x^2 + y^2 = 4$ and the xy-plane. (14 points)