

# King Fahd University of Petroleum and Minerals

First Major Exam for Math 202

**Time allowed 1 hour and 30 minutes**

Full Name: .....

ID Number: .....

Section: .....

**Note** The following things are prohibited

- Using an advanced calculator
- Having the mobile phone on
- Talking to each other
- Cheating

Major 1

**Problem 1** (10 Points) Consider the following DE

$$(1 + x^2)\frac{dy}{dx} + 2xy = \frac{x^2}{1 + x^2}. \quad (1a)$$

- i- Is the DE (1a) linear or nonlinear?
- ii- Write the DE (1a) in the standard form  $\frac{dy}{dx} + P(x)y = f(x)$ .
- iii- Find the integrating factor  $e^{\int P(x)dx}$ .
- iv- Solve the DE (1a).

**Problem 2** (8 Points) Consider the following DE

$$y\sqrt{1 - x^2}\frac{dy}{dx} = e^{-y}x. \quad (1b)$$

- i- Write the DE (1b) in the form  $g(y)dy = f(x)dx$ .
- ii- Solve the DE (1b) subject to the initial condition  $y(0) = 2$ .
- iii- Find the largest interval I on which the solution is defined.

**Problem 3** (12 Points) Consider the following DE

$$2y^2dx + x(x - y)dy = 0. \quad (1c)$$

- i- Show that the DE (1c) is homogeneous.
- ii- State all possible types of substitutions that allowed you to transform the DE (1c) into a separable equation.
- iii- Let  $y = ux$  where  $u$  is a function of  $x$ . By substituting  $y$  and  $dy$  in the DE (1c) show that

$$\frac{u - 1}{u + u^2} du = \frac{dx}{x}. \quad (*)$$

- iv- Solve the DE (\*)
- v- Using the above information, find the solution of the DE (1c)

**Problem 4** (10 Points) Consider the following IVP

$$y(y + \sin x) dx + (2xy + \ln y^2 - \cos y) dy = 0, \quad y(0) = 1. \quad (1d)$$

- i- Verify that the DE defined in (1d) is exact.
- ii- Solve the IVP (1d).

**Note:** You may need to use integration by parts in the last part.