King Fahd University of Petroleum and Minerals

Final Exam for Math 202

Time allowed 2 hour and 30 minutes

Full Name: ....

ID Number: ....

Section: ....

Note The following things are prohibited

- Using an advanced calculator
- Having the mobile phone on
- Talking to each other
- Cheating

Problem 1 (8 Points) Use the method of undetermined coefficient (annihilator) to solve the DE:

$$y''' - 2y'' - y' + 2y = 4e^{-2x} + 2.$$

Solution

Problem 2 (8 Points) Verify that

$$\phi(t) = C_1 \begin{pmatrix} 6 \\ -1 \\ -5 \end{pmatrix} e^{-t} + C_2 \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} e^{-2t} + C_3 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} e^{3t}$$

forms the general solution of the homogeneous system of DEs:

$$X' = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} X$$

on $(-\infty, \infty)$.

solution

Problem 3 (10 Points) The function $y_1 = \sqrt{x}\cos x$ is a solution of the DE:

$$x^2y'' - xy' + (x^2 + \frac{3}{4})y = 0$$

on $I = (0, \infty)$.
i- Find a second solution y_2 such that y_1, y_2 form a fundamental set of solutions of the given DE on I.

ii- Use the method of variation of parameters to find the particular solution y_p on I of the nonhomogeneous DE:

$$x^2y'' - xy' + (x^2 + \frac{3}{4})y = x^{5/2}.$$

Problem 4 (12 Points) Consider the following nonhomogeneous system of DEs:

$$X' = AX + F(t) \tag{1}$$

where

$$A = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} \quad \text{and} \quad F(t) = \begin{pmatrix} -9 \\ 0 \end{pmatrix} e^t.$$

i- Show that the matrix A has a repeated eigenvalue with an order of multiplicity 2.

ii- Find the general solution for the homogeneous system of DEs: $X' = AX$.

iii- Use the method of undetermined coefficient to solve (1).

Solution

Problem 5 (12 Points) Let A be a 3×3 matrix defined by

$$A = \begin{pmatrix} -1 & 2 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

i- Find the eigenvalues of A.

ii- Find the corresponding eigenvectors of A.

iii- Find the general solution for the homogeneous system of DEs: $X'(t) = AX(t)$ in both complex and real forms.

Solution

Problem 6 (14 Points) Consider the following DE

$$2xy'' + y' - 2y = 0.$$

i- Find the regular singular point(s).

ii- Let $y = \sum_{n=0} c_n x^{n+r}$. Show that

$$r(2r-1)c_0 = 0 \quad \text{and} \quad c_{k+1} = \frac{2c_k}{(k+r+1)(2k+2r+1)} \quad \text{for} \quad k \geq 0.$$

iii- For $r = \frac{1}{2}$, use the recurrence relation in the part (ii) to find c_k for $k \geq 1$ in terms of c_0.

iv- Using the above information, find a power series solution of the given DE about 0.

v- For $r = 0$, use the recurrence relation in the part (ii) to find c_k for $k \geq 1$ in terms of c_0.

Solution