KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Final Exam

Math 202
(Elements of Differential Equations)

Time Allowed: 2 Hours

Student Name:_______________  Id. No. _______________
Section:_______________

Note

No programmable calculators and mobile phones allowed in the examination hall. For all questions show calculations in support of your answers.

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Q1 Solve the IVP \(2x \frac{dy}{dx} = 2y + xe^{2y/x}\) such that \(y(1) = 1\).
Q2 Give general solution of \( D^3 (D^2 - 1)^2 (D^2 + 4)^3 y = 0 \)
Q3 Find general solution of the ODE \[ \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} = e^{2x} + \sin x \]
Q4 Let “P(t)” be the population of a city at any time “t”. Assume that the rate of change of the population with time is directly proportional to the population “P(t)”. Construct the differential equation that governs this model and give general solution.
Q 5  Find two independent series solutions of $\frac{d^2 y}{dx^2} - 4xy = 0$ about ordinary point $x = 0$. 
Q 6 Solve the system \( x^t = \begin{pmatrix} 6 & -1 \\ 4 & 2 \end{pmatrix} x \)
Q 7 Solve the homogeneous system $x' = \begin{pmatrix} 2 & 2 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} x$. Use following hint:

(One Eigen value $\lambda = 1$ and corresponding Eigenvector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$)
Q 8  Find particular solution of the non-homogeneous system $x' = \begin{pmatrix} 1 & 8 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 12t \\ 12t \end{pmatrix}$

Hint:

(1): Two Eigenvalues of the matrix A are: 3, -3

(2): Eigenvectors with respect to Eigenvalue 3 is: $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

(3): Eigenvector with respect to Eigenvalue -3 is: $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$