KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Department of Mathematical Sciences

Math 301                     Method of Applied Mathematics

Final Examination                                                   Term 061

Time Allowed    3 Hours

Name _________________      ID #   _______            Section #

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Important Note

Show all work.
Use of programmable calculator is not allowed.
Mobiles and paging devices should not be carried during examination.

Instructor: F. D. Zaman
Q1)(a) Use the **Stokes theorem** to evaluate the line integral \( \int_C F \cdot dr \) where

\[ F = zi + xj + yk \]

and \( C \) is boundary of \( S: \) plane \( x + y + z = 1 \) bounded by the coordinate planes. (4)
Q1)(b) Use the divergence theorem to find the flux integral \( \iint_S \mathbf{F} \cdot \mathbf{n} \, ds \) where
\[
\mathbf{F} = e^y \mathbf{i} + xz^2 \mathbf{j} + (z - 1)^2 \mathbf{k},
\]
\( S \) being surface of \( x^2 + y^2 = 16 \) between \( z = 1 \) to \( z = 4 \).  

(3)
Q 2) Solve the following initial value problem using the Laplace transform

\[ y'' - 5y' + 6y = e' u(t-1), \]
\[ y(0) = 0, \ y'(0) = 1. \]
Q 3) Solve the Laplace equation using separation of variables

\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 1, \quad 0 < y < 1, \]

\[ u(0, y) = 0, \quad u(1, y) = 0, \quad 0 < y < 1, \]

\[ u(x, 0) = 0, \quad u(x, 1) = 10, \quad 0 < x < 1. \]
Q 4)(a) Find the temperature $u(r, t)$ in a circular plate by solving the following problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} = \frac{1}{k} \frac{\partial u}{\partial t}, \quad 0 < r < 1$$

$u(1, t) = 0, \quad t > 0$

$u(r, 0) = f(r), \quad 0 < r < 1.$
Q 4)(b) Use the Fourier Bessel series to write the solution obtained in part (a) for $f(r) = 1$. (3)
Q 5) Use the Fourier transform to solve the following boundary value problem

\[
\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, \; t > 0
\]

\[
u(x,0) = f(x) = \begin{cases} 
0, & x < -1 \\
-1, & -1 < x < 0 \\
1, & 0 < x < 1 \\
0, & x > 1.
\end{cases} \quad (7)
\]
Q 6) Find the solution to the following problem using an appropriate integral transform

\[ \frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2} \frac{\partial^2 u}{\partial t^2}, \quad x > 0, t > 0 \]

\[ \frac{\partial u}{\partial x}(0, t) = 0, \quad t > 0 \]

\[ u(x, 0) = e^{-x} \]

\[ \frac{\partial u}{\partial t}(x, 0) = 0. \]