MATH 302 -01 (Term 061)

Major Exam II

Monday 20 November, 2006

Time Allowed: 75 minutes

Instructor: Dr. A. Boucherif

Name:..........................................................ID #............................

Serial #:............

• Write clearly.
• Show all your steps.
• No credit will be given to wrong steps.
• Do not do messy work
• Calculators are not allowed in this exam.

Marks. Q1: 2 points-Q2: 4 points-Q3: 5 points-Q4: 6 points-Q5: 3 points/ 

Total Mark =
Q1. Compute the maximum and the minimum rate of change of 
\[ \varphi(x, y, z) = \frac{1}{\|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\|} \] at the point \( P = (0, 1, 1) \).
Q2. Consider the curve $C$ with parametric equations $x = e^{-t}$, $y = 1 + 2 \sin t$, $z = t - \cos t$, $t \geq 0$. The value $t = 0$ corresponds to a point $P_0$ on the curve.

I. Circle the correct answer.

(i) the unit tangent vector $T$ at $P_0$ to the curve is:

(a) $T = i + j - k$, \hspace{1cm} (b) $T = -i + 2j - k$,

(c) $T = \frac{-i + 2j + k}{\sqrt{6}}$, \hspace{1cm} (d) $T = \frac{i + 2j + 2k}{\sqrt{6}}$

(ii) The directional derivative of $\varphi (x, y, z) = x^2yz^3$ in the direction of $T$ is:

(a) $-6$, \hspace{1cm} (b) $-3$, \hspace{1cm} (c) $\frac{\sqrt{6}}{2}$, \hspace{1cm} (d) $\frac{2}{\sqrt{6}}$

II. Justify your choices.
Q3. Evaluate the line integral from (-1,0) to (1,0) in the xy-plane of the vector field $\mathbf{F}(x, y) = yi + xj$ along
   (i) the x-axis,
   (ii) the semicircle $y = \sqrt{1 - x^2}$,
   (iii) using Green’s Theorem.
Q4. Let \( F = \frac{y}{x} i + (\ln x) j \) and let \( C \) be any path connecting \( P_0 = (1, 1) \) to \( P_1 = (2, 2) \).

(i) Show that \( F \) is conservative.
(ii) Find \( \varphi \) so that \( F = \nabla \varphi \)
(iii) Evaluate \( \oint_C F \cdot dR \).
Q5. Evaluate \( \int \int (x^2 + y^2) \, d\sigma \) where \( \sum \) is the surface of the cone \( z^2 = 3(x^2 + y^2) \) bounded by \( z = 0 \) and \( z = 3 \).