King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

**CODE 001** | **Math 102** | **CODE 001**

# Math 102
## Final Exam
## Thursday 7/6/2007
## Net Time Allowed: 165 minutes

Name: ______________________________

ID: ____________________ Sec: ____________________.

**Check that this exam has 25 questions.**

## Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The region bounded by the curves $y = 4x - x^2$ and $y = 8x - 2x^2$ is rotated about the line $x = -4$. Then the volume of the resulting solid is given by

(a) $2\pi \int_0^4 (x+4)(x^2 - 4x)dx$

(b) $2\pi \int_{-4}^4 (x+4)(x^2 - 4x)dx$

(c) $2\pi \int_0^4 (x-4)(x^2 - 4x)dx$

(d) $2\pi \int_0^4 (x-4)(4x - x^2)dx$

(e) $2\pi \int_0^4 (x+4)(4x - x^2)dx$

2. The length of the curve $y = \frac{2}{3}(x^2 - 1)^{3/2}, \quad 1 \le x \le 3$ is equal to

(a) $\frac{15}{4}$

(b) $\frac{46}{3}$

(c) 4

(d) 15

(e) $\frac{22}{3}$

3. The improper integral $\int_0^1 \ln(2x)dx$ is

(a) equal to $+\infty$

(b) convergent and has the value $-1 + \ln 2$

(c) equal to $-\infty$

(d) convergent and has the value $1 + \ln 2$

(e) convergent and has the value $1 - \ln 2$

4. $\int_0^{\ln\sqrt{3}} \frac{1}{e^x + e^{-x}} dx =$

(a) $\frac{\pi}{4}$

(b) $\sqrt{3} - 1$

(c) $\frac{\pi}{12}$

(d) $\frac{\pi}{3}$

(e) $e - 1$

5. The area between the curves $y = \sin 2x$ and $y = \cos x$ from $x = 0$ to $x = \dfrac{\pi}{2}$ is equal to

(a) $0$

(b) $1$

(c) $\dfrac{1}{2}$

(d) $\dfrac{1}{4}$

(e) $\sqrt{3} - \dfrac{3}{2}$

6. The indefinite integral $\displaystyle\int_0^{\pi/4} \frac{\sin 3x}{\cos x}\, dx$ is equal to

(a) $1 + \ln \sqrt{2}$

(b) $1 + \ln 2$

(c) $1 + \sqrt{2}$

(d) $1 - \ln 2$

(e) $1 - \ln \sqrt{2}$

7. By recognizing the sum as a Riemann sum for a function defined on $[0,1]$, the value of the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} e^{-i/n}$ is

(a) $0$

(b) $e-1$

(c) $1$

(d) $-3$

(e) $1-e^{-1}$

8. The integral $\int \frac{dx}{x+x^{3/2}}$ is equal to

(a) $\ln x - 2\ln(\sqrt{x}+1) + C$

(b) $\ln x - \ln\sqrt{x} + 1 + C$

(c) $\ln x + 2\ln(\sqrt{x}+1) + C$

(d) $\ln x - 2\ln(\sqrt{x}-1) + C$

(e) $\ln x + \ln(\sqrt{x}+1) + C$

9. The value of $\int_0^1 x^3\sqrt{1-x^2}\,dx$ is equal to

(a) $\dfrac{2}{15}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{4}{15}$

(d) $\dfrac{1}{5}$

(e) $\dfrac{8}{15}$

10. The integral $\int \dfrac{dx}{1-\sin x}$ is equal to

(a) $\cot x + \csc x + C$

(b) $\tan x - \sec x + C$

(c) $\tan x + \csc x + C$

(d) $\tan x + \sec x + C$

(e) $\sec x - \tan x + C$

11. The average value of $f(x) = \sqrt{9 - x^2}$ on $[0, 3]$ is equal to

(a) $\frac{10\pi}{4}$

(b) $\frac{3\pi}{16}$

(c) $\frac{3\pi}{4}$

(d) $\frac{9\pi}{16}$

(e) $3\pi$

12. If $F(x) = \int_3^{x^2} \frac{\tan^{-1}\sqrt{t}}{\sqrt{t}}\, dt,\ \ x > 0$, then $8F(\sqrt{3}) + 9F'(\sqrt{3}) =$

(a) $6\pi$

(b) $8\pi$

(c) $17\sqrt{3}\pi$

(d) $3\pi$

(e) $\sqrt{3}\pi$

13. The improper integral $\int_e^\infty \frac{dx}{x(\ln x)^2}$

(a) diverges

(b) converges to 0

(c) converges to $\frac{1}{e}$

(d) converges to $e$

(e) converges to 1

14. Consider the series $\sum_{n\geq 2} \frac{\cos^2 n}{n^2+2n+1}$

(a) The series diverges

(b) The series converges by alternating series test

(c) The series converges and its sum is zero

(d) The series converges and its sum is less than $\frac{1}{2}$

(e) The series converges with sum more than or equal to $\frac{1}{2}$

15. The series $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n!}$

(a) converges conditionally

(b) converges absolutely

(c) is convergent to 0

(d) is convergent to $\frac{1}{e}$

(e) is divergent

16. The radius and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$ are respectively

(a) $\frac{1}{2}$ and $\left(\frac{5}{2}, \frac{7}{2}\right)$

(b) 1 and $[2, 4)$

(c) $\frac{1}{2}$ and $\left[\frac{5}{2}, \frac{7}{2}\right)$

(d) 1 and $(2, 4)$

(e) $\frac{1}{2}$ and $\left(\frac{5}{2}, \frac{7}{2}\right]$

17. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$

(a) converges for $p \leq 0$

(b) converges for all real numbers $p$

(c) diverges for all $p$

(d) converges only for $p = 0$

(e) converges for $p > 0$

18. The limit of the sequence $\{n\sqrt[n]{e} - n\}_{n=1}^{+\infty}$

(a) is equal to 1

(b) is equal to 0

(c) is equal to $e$

(d) does not exist

(e) is equal to $-2$

19. The series $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$

(a) diverges by the test for divergence

(b) diverges by comparison test

(c) converges

(d) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$

(e) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$

20. The series $\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^{-k}$

(a) converges to $e$

(b) diverges

(c) converges to $\frac{1}{e}$

(d) converges to 0

(e) converges to 1

21. The value of $a$ for which the series $\sum_{n=0}^{\infty} 4^n (3+a)^{-n}$ converges to 2 is equal to

(a) 5

(b) 1

(c) 3

(d) 0

(e) 6

22. If you want to use the integral test to test the series $\sum_{n=1}^{\infty} n\, e^{-n^2}$ for convergence, then your conclusion is

(a) the integral test is not applicable in this case

(b) the integral converges to $\dfrac{1}{2e}$

(c) the integral converges to $3e$

(d) the integral diverges

(e) the integral converges to $\dfrac{1}{e^2}$

23. For $x > 0$, the series $\sum_{n=0}^{\infty} \frac{(-2)^n (\ln x)^n}{n!}$ converges to
[Hint: Use the Maclaurin series of $e^x$]

(a) $x$

(b) $e^x$

(c) $\frac{1}{x}$

(d) $\frac{1}{x^2}$

(e) $x^2$

24. An integral for the area of the surface obtained by rotating the curve $y = \sec x$,
$0 \le x \le \frac{\pi}{4}$ about the $y$-axis is

(a) $\int_0^{\pi/4} 2\pi y \sqrt{1 + (\sec^{-1} x \tan^{-1} x)^2}\, dx$

(b) $\int_0^{\pi/4} 2\pi \sec^{-1} y \sqrt{1 + \frac{1}{y^2(y^2+1)}}\, dy$

(c) $\int_1^{\sqrt{2}} 2\pi y \sqrt{1 + \frac{1}{y^2(y^2-1)}}\, dy$

(d) $\int_1^{\sqrt{2}} 2\pi x \sqrt{1 + (\sec x \tan x)^2}\, dx$

(e) $\int_0^{\pi/4} 2\pi x \sqrt{1 + (\sec x \tan x)^2}\, dx$

25. A power series representation for $f(x) = \dfrac{3x^3}{(x-3)^2}$ is given by

(a) $\sum_{n=1}^{\infty} \dfrac{x^{n+3}}{3^n}$

(b) $\sum_{n=1}^{\infty} \dfrac{n}{3^{n+2}} x^n$

(c) $\sum_{n=1}^{\infty} n \left(\dfrac{x}{3}\right)^n$

(d) $\sum_{n=1}^{\infty} \dfrac{n+2}{3^n} x^n$

(e) $\sum_{n=1}^{\infty} \dfrac{n}{3^n} x^{n+2}$