

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Semester II, 2006-2007(062)
MATH 201 Sec. 7
Final Exam

Student Name: _____

Student ID: _____

Section: _____

Note:

FOR ALL PROBLEMS, SHOW WORK. NO CREDIT FOR ANSWERS NOT SUPPORTED BY WORK.

1. (a) Let $P_1 = (1, 1, 1)$, $P_2 = (1, -1, 1)$, $P_3 = (0, 0, 1)$ and $P_4 = (1, 1, 0)$. Find the projection of $\overrightarrow{P_1P_2}$ on the normal to the plane determined by P_2, P_3, P_4 .
- (b) Using (a), find the distance of P_1 from the plane determined by P_2, P_3 and P_4 .

2. Identify and draw a rough sketch of the surfaces with the given equations:

(a) $x^2 + 4y^2 + 9z^2 = 1$.

(b) $x^2 - y^2 + z^2 = 1$.

(c) $y^2 = x^2 + 2z^2$.

(d) $x^2 + 2z^2 = 1$.

3. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{x^2 + y^2}$ does not exist.

4. If $z = f(x, y)$, and $x = r^2 + s^2$ and $y = 2rs$, find $\frac{\partial^2 z}{\partial r \partial s}$.

5. Find points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ where the tangent plane is parallel to the plane $3x - y + 3z = 1$.

6. Find local maxima, minima and saddle points – if any – of $f(x, y) = 1 + 2xy - x^2 - y^2$.

7. Sketch the region of integration and change the order of integration:

$$\int_0^4 \int_0^{\sqrt{x}} f(x, y) dy dx.$$

8. Use polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 1$.

9. Evaluate $\iiint_E z \, dV$, where E is bounded by the paraboloid $z = 4x^2 + 4y^2$ and the plane $z = 4$.

10. Use spherical coordinates to evaluate $\iiint_E z \, dV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

11. Use spherical coordinates to show

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{x^2 + y^2 + z^2} e^{-(x^2+y^2+z^2)} dx dy dz = 2\pi.$$