King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
**Math 201 (Section 8)**
**Final Examination**
**Semester II, 2006–2007(062)**
**June 07, 2007**
**Time: 07:30 a.m. – 10:30 a.m.**

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Name: ______________________________

ID #: ____________________ Serial #: ______________

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| Instructor | Location |
|---|---|
| Dr. Abdul Rahim Khan | Building 54 (Exhibition Center) |

**Instructions**:

1. Do not use programmable calculators. Use of ordinary calculator is allowed.
2. Show all your work. Less credit will be given for answers not supported by proper work.
3. This exam consists of $\boxed{13}$ pages.
4. Do not forget to write your NAME, ID, and Serial # in the space provided above.

| Question # | Grade/Points |
|---|---|
| 1 | ____________/18 |
| 2 | ____________/18 |
| 3 | ____________/18 |
| 4 | ____________/18 |
| 5 | ____________/18 |
| 6 | ____________/20 |
| Total: | ____________/110 |

1. (a) For the polar curve, $r = 1 - \cos\theta$, find

   (i) singular point(s) (5 points)

   (ii) arc length. (4 points)

(b) Calculate area of the region that lies inside $r = 2 - 2\cos\theta$ and outside $r = 1$.
(9 points)

2. (a) Find an equation of the set $S$ of all points equidistant from the points $A(-1, 5, 3)$ and $B(6, 2, -2)$. Hence describe the set $S$ geometrically. (9 points)

(b) Find equation of a plane that contains the point $P(5, 0, 2)$ and the line $x = 1 + 3t, y = 4 - 2t, z = -3 + t.$ (9 points)

3. (a) Describe the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = -4$. Draw its rough sketch.
(9 points)

(b) Show that $u = \sin(x - at) + \ln(x + at)$ is a solution of the wave equation $u_{tt} = a^2 u_{xx}$. (9 points)

4. (a) Find equation of the normal line at $(-2, 1, -3)$ to ellipsoid $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{9} = 3$.
(5 points)

(b) Locate extrema and saddle points of $f(x, y) = x \sin y$. (4 points)

(c) Use the Lagrange multiplier method to find extrema of $(x, y, z) = z - x^2 - y^2$ subject to the constraints $x + y + z = 1$ and $x^2 + y^2 = 4$. (9 points)

5. (a) Evaluate: $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy.$ (9 points)

(b) Use polar coordinates to find volume of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the $xy$-plane. (9 points)

6. (a) Use an iterated integral to find volume of the solid that is bounded by the parabolic cylinders

$$z = x^2, y = x^2 \text{ and } y = 8 - x^2.$$

(10 points)

(b) Change the triple integral to cylindrical coordinates (do not evaluate it):

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)dz\,dx\,dy.$$

(5 points)

(c) Set up a triple integral in spherical coordinates to find volume of the solid within the sphere $x^2+y^2+z^2=9$, outside the cone $z=\sqrt{x^2+y^2}$ and above the $xy$-plane. (5 points)