Questions for review on Math 202

Elements of Differential Equations

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1. State what is meant by Differential Equations.

2. Do you know any application for Differential Equations; give some examples.

3. Write a brief classification with examples of the types of DEs that you studied in your course Math 202.

4. Does every differential equation have a solution.

5. If we know a solution for a given DE, is it necessarily to be unique?

6. What do we mean by an initial value Problem?

7. What do we mean by Cauchy-Euler differential equation? Give an example and show how to solve such type of equations.

8. Complete the following table

<table>
<thead>
<tr>
<th>Equation</th>
<th>Order</th>
<th>Linear / Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y' = 10 + y^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 \frac{dy}{dx} + 5xy = 0 )</td>
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</tr>
<tr>
<td>( y = 2xy' + y(y')^2 )</td>
<td></td>
<td></td>
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<tr>
<td>( y'' + y = \tan x )</td>
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<tr>
<td>( y'' - 5y' + 6y = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y' + 3x(y'')^3 = \sin x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y' + 3\sin x y'' = \cos x )</td>
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</tbody>
</table>

9. Classify the following 1st Order ODE as Separable, Linear in \( y \) (or in \( x \)), Homogeneous (with its degree), Bernoulli, or Exact.

i. \((y + y^2)dx - (x + x^2)dy = 0\)

ii. \((y - xy^2)dy = ydx\)

iii. \((e^{y/x} + e^{x/y} + 1)dy = (1 + \ln(y/x))dx\)

iv. \(\frac{dy}{dx} = \sqrt{x^2 - y^2}\)

v. \(3\frac{dy}{dx} = 4x - y\)
10. Solve \( x^2 \frac{dy}{dx} = y - xy \)

11. Solve \( x \frac{dy}{dx} - y = x^2 \sin x \)

12. Solve the initial value problem \((e^x + y)dx + (2 + x + ye^x)dy = 0, \ y(0) = 1.\)

13. Solve the initial value problem \( \frac{dy}{dx} = \cos(x + y), \ y(0) = \pi/4 \)

14. Solve \( x \frac{dy}{dx} - (1 + x)y = xy^2 \)

15. Solve \( (y^2 - xy)dx + x^2 dy = 0 \)

16. Is \( y = xe^{-2x} \) a solution to \( y'' + 4y' + 4y = 0 ? \)

17. How many solutions are there to the initial value problem \( \frac{1}{x^2} \frac{dy}{dx} + y^2 = \frac{1}{x}, \ y(0) = 2. \) Justify your answer.

18. The Population of a Community is known to increase at a rate Proportional to the number of People present at any time. The Population of the community is doubled after 5 years and it is 10,000 after 3 years. What was the initial population. What will be the Population after 10 years.

19. If \( y_1 = \ln x \) is a solution of the equation \( xy'' + y' = 0, \) use reduction of order Or an appropriate formula to find a second solution.

20. Solve the boundary value problem: \( y'' - 10y' + 25y = 0, \ y(0) = 1, \ y(1) = 0.\)

21. Find the general solution of the following Cauchy-Euler Equation \( 2x^2 y'' + 5xy' + y = 0 \)

22. Find the solution of the BVP \( y^{(4)} + y'' = 0 \) satisfying the conditions:
\( y(0) = 0, \ y'(\pi) = 0, \ y'(0) = 1, \ y''(\pi) = -1 \)

23. Write a homogeneous linear differential equation whose auxiliary equation is \( 5m^2 - 2m^3 + 4m = 0 \)

24. Given \( y_1 = x \sin(linx) \) a solution of the DE \( x^2 y'' - xy' + 2y = 0. \) Find another solution for this equation.
25. Using Wronskian show that the functions $1, 1/x$ and $\log x$ are linearly independent on the interval $(0, \infty)$.

26. Show that $1, x, \sin x, \cos x$ form a **Fundamental Set of the solutions** of the Differential Equation $y^{(4)} + y'' = 0$ on $(-\infty, \infty)$.

27. Use the method of **Variation of Parameters** to find the general solution of the differential equation $\frac{d^2y}{dx^2} + y = \sin x$.

28. Solve the above question using the method of **Undetermined Coefficients**.

29. Solve the DE: $y''' - xy'' = 8x^2$.

30. If $y_p = u_1y_1 + u_2y_2 + u_3y_3$ is a particular solution of $y^{(3)} + 9y^{(1)} = \tan x$, then find: (i) $y_1, y_2, y_3$ (ii) $u_1', u_2', u_3'$

31. Find all Singular Points of the ODE and classify them as regular or irregular singular point: $x^3(x^2 - 9)y'' - 2x^2(x + 3)y' + (x - 3)y = 0$.

32. Use the **Power Series method** to find the General solution of the DE $y'' - 4xy' - 4y = e^x$ about $x_0 = 0$.

33. Show that $x_0 = 0$ is a regular singular point of the differential equation $(6x + 2x^3)y'' + 21xy' + 9(x^2 - 1)y = 0$.

Then find the **Indicial Equation** and its roots about $x_0 = 0$.

34. Use **Gauss-Jordan Elimination Method**, to solve the system:

$$
\begin{align*}
    s - t + u + v &= 0 \\
    2s + 2u &= 0 \\
    s + t + u - v &= 0 \\
    -s - 3t - u + 3v &= 0
\end{align*}
$$

35. Find the inverse of $A$, if it exists, where

$$
A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}
$$

36. Find the **eigen values** of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, and find the corresponding **eigen vectors**.
37. Solve the system
\[
\begin{align*}
\frac{dx}{dt} &= x \\
\frac{dy}{dt} &= 2x + 2y - z \\
\frac{dz}{dt} &= y
\end{align*}
\]

38. Solve the system
\[
\begin{align*}
\frac{dx}{dt} &= 3x + 4y \\
\frac{dy}{dt} &= -4x + 3y
\end{align*}
\]

39. Solve the system
\[
X' = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 0 \\ 6 & 3 & -8 \end{bmatrix} X
\]

40. Solve the following non homogeneous system
\[
X' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ te^t \\ e^t \end{bmatrix}
\]