Important instructions:
- Use an HB pencil or a pen (do not use red color)
- Solve the problems completely
- Write down your answers in a clear manner
- Justify all your steps
- Use the back of the page (verso) only for scratching
Prob. 1

Let $T(x, y, z) = x^2 + y^2 + z^2$ represent temperature and let the flow of heat be given by the vector field $\vec{F} = -\vec{\nabla}T$. Find the flux of heat out of the sphere $x^2 + y^2 + z^2 = a^2$. 
**Prob. 2**

Use the Laplace transform to solve the differential equation

\[ y'' - 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 3 \]
Prob. 3
Use the Laplace transform to solve the integral equation

\[ f(t) = 1 + t - \frac{8}{3} \int_0^t (\tau - t)^3 f(\tau) d\tau \]
Prob. 4
Solve the initial value problem
\[ y'' - 7y' + 6y = e^t + \delta(t - 2) + \delta(t - 4), \quad y(0) = 0, \quad y'(0) = 0. \]
Prob. 5
The root-mean-square value of a function $f(x)$ defined over an interval $(a, b)$ is given by

$$RMS(f) = \sqrt{\frac{\int_a^b f^2(x) \, dx}{b-a}}.$$ 

Show that on the interval $(-p, p)$ we have

$$RMS(f) = \sqrt{\frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)}$$

where $a_n$ and $b_n$ are the Fourier coefficients.