KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Department of Mathematics and Statistics

Math 302                  Engineering Mathematics
Final Exam                                                        Term 062

Time Allowed 3 Hours

Name __________________                  ID #  _______

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Important Note

Show all work.
Use of programmable calculators is not allowed.
Mobiles and paging devices should not be carried during examination.

Instructor: F. D. Zaman
Q 1) (a) Given \( \vec{F} = \sin z \hat{i} + e^z \hat{j} + zk \), use Gauss divergence theorem to evaluate the flux integral through surface of cylinder centered at origin with radius 3 for \( 0 \leq z \leq 2 \).
Q 1) (b) Evaluate \[ \int_C \mathbf{F} \cdot d\mathbf{R} \] where
\[ \mathbf{F} = (3y^2 + 3\sin y + 4)i + (6xy + 3x \cos y)j \] along any path joining (0,0) to \((3, \pi / 2)\).
Q2(a) Find eigenvalues and eigenvectors of the following matrix
\[
\begin{bmatrix}
-2 & 2 & -3 \\
2 & 1 & -6 \\
-1 & -2 & 0
\end{bmatrix}
\]
Q2)(b) For the matrix in part A, give the diagonalizing matrix, if it is diagonalizable. Give the diagonal form of the matrix.
Q3)(a) Find the series expansion for \( f(z) = \frac{1}{z + 3} \) about \( 1 - 4i \).
Q3)(b) Find all solutions of \( iz^2 - (1 + i)z + 2 = 0 \).
Q3)(c) Determine all values of
(i): \( 4^{-2-3i} \)

(ii) \( \log(2 + 2i) \)
Q4 Evaluate the following integrals:

(i): \[ \oint \frac{z^2 \sin \left( \frac{1}{z} \right)}{z} \, dz \]

\[ \Gamma : |z| = 1. \]
(ii) \[ \oint_C \frac{dz}{z^2 (z^2 + 9)} \]

\( \Gamma \): circle centre at \( z = 3i \) with radius 4.
Q5 Evaluate the real integral:
\[ \int_{0}^{2\pi} \frac{d\vartheta}{(3 - 2\cos \vartheta)} \]
Q6 Evaluate the following improper integrals:

(i): \[ \int_{\infty}^{\infty} \frac{x^2}{-x^4 + 1} \, dx \]
(ii) \[ \int_{-\infty}^{\infty} \frac{\cos x}{4x^2 + 1} \, dx \]