King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 311

Second Major Exam

Semester II, 2006-2007 (062)

April 22, 2007

8:30 p.m. – 10:00 p.m.

Name: ____ Grade ____________/40

ID#: ____

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1. Show all your work. Less credit will be given for answer not supported by proper work.
2. Clearly indicate the theorem or result you use.
3. This exam consists of 9 pages.

1. Suppose that $I_n = [a_n, b_n]$ is a sequence of closed intervals such that $I_{n+1} \subseteq I_n$ for each n. If $\lim_{n\to\infty} (b_n - a_n) = 0$, then prove that there is one and only one x_0 that belongs to all I_n.

(5 points)

2. Prove that any bounded infinite sequence of real numbers has a convergent subsequence.

(5 points)

3. Suppose that f is continuous on the closed interval $I = [a, b]$. Show that f takes maximum and minimum values on I (that is, there exist numbers x_0 and x_1 in I such that $f(x_0) \leq f(x) \leq f(x_1)$ for all $x \in I$).

(5 points)

4. Define a uniformaly continuous function on $D \subseteq \mathbb{R}$. Show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0,1]$.

(5 points)

5. (a) Show that $x_n = \left(\dfrac{\cos\, n\pi}{n} \right)$ is a Cauchy sequence.

(2 points)

(b) Prove that any Cauchy sequence of real numbers is convergent.

(3 points)

6. Let $f(x) = \begin{cases} x\sin\frac{1}{x} & 0 < x \leq 1 \\ 0 & \text{x = 0} \end{cases}$

Show that $f(x)$ is uniformly continuous on $[1, \infty)$.

(5 points)

7. Suppose that a family F of open intervals covers the closed interval $I=[a,b]$. Then prove that a finite subfamily of F covers I.

(5 points)

8. (a) State intermediate value theorem

(2 points)

(b) Let $F = \left\{ \left(\frac{-2}{n}, \frac{2}{n} \right) : n \in \mathbb{N} \right\}$.

Check whether or not F covers $A = [0,1]$.

Does F have a finite subcover for A?

Is A a compact set?

(3points)