

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematical Sciences**  
**Semester II, 2006-2007(062)**  
**MATH 430**  
**Major I**

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Student Name: \_\_\_\_\_

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1. Find the sum of the series

$$1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta.$$

2. Use  $e^{i\theta} = \cos \theta + i \sin \theta$  to evaluate  $\int_0^{2\pi} \sin^5(2\theta) d\theta$ .

3. Solve the equation

$$(z + 1)^5 = z^5.$$

4. Let  $J(z) = \frac{1}{2} \left( z + \frac{1}{z} \right)$ . Show that  $J$  maps the unit circle  $|z| = 1$  onto the interval  $[-1, 1]$  and the circle  $|z| = r$  ( $r \neq 0$ ) to the ellipse  $\frac{u^2}{\left[ \frac{1}{2} \left( r + \frac{1}{r} \right) \right]^2} + \frac{v^2}{\left[ \frac{1}{2} \left( r - \frac{1}{r} \right) \right]^2} = 1$ .

5. Show that

$$f(z) = e^{x^2 - y^2} [\cos(2xy) + i \sin 2xy]$$

is differentiable everywhere and find  $f'(z)$ .

6. Show that if  $f$  is analytic and  $\operatorname{Re} f$  is constant, then  $f$  is also constant.