

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Semester II, 2006-2007(062)
MATH 430
Major Exam II

Name: _____ ID #: _____

1. If $p(z) = a_n z^n + \cdots + a_1 z + a_0$ ($a_n \neq 0$), define $p^*(z)$ by

$$p^*(z) = \overline{a_n} + \overline{a_{n-1}}z + \cdots + \overline{a_0}z^n.$$

- (a) Show that $p^*(z) = z^n \overline{p(1/\overline{z})}$.
(b) Show that if $|z| = 1$, then

$$|p(z)| = |p^*(z)|.$$

2. Define $\sin z$ and $\cos z$ and verify that $\sin z - \sin w = 2 \cos \left(\frac{w+z}{2} \right) \sin \left(\frac{w-z}{2} \right)$.

3. Solve the equation

- (a) $\log(z^2 - 1) = i\frac{\pi}{2}$.
(b) $e^{2z} + e^z + 1 = 0$.

4. Find all solutions of

$$\sin z = \cos z.$$

5. Compute $\int_{\Gamma} \overline{z} dz$. where

- (a) Γ is the circle $|z| = 2$ traversed once counterclockwise
(b) traversed three times clockwise.

6. Use $\left| \int_{\Gamma} f(z) dz \right| \leq M \text{ length } \Gamma$, where $M = \max$ of $|f(z)|$ on Γ to show:

- (a) $\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{3\pi}{4}$, where C is the circle $|z| = 3$ traversed once.

- (b) if γ is the vertical line segment from $z = R$ to $z = k + 2\pi i$ then $\left| \int_{\gamma} \frac{e^{3z}}{1 + e^z} dz \right| \leq \frac{2\pi e^{3R}}{(e^R - 1)}$.