

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematics and Statistics**  
**Semester II, 2006-2007(062)**  
**MATH 430**  
**Final Exam**

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Name: \_\_\_\_\_ ID #: \_\_\_\_\_

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1. For the function  $f(z) = e^z$  (10 points)
  - (a) Describe the domain and range.
  - (b) Show that  $f(-z) = \frac{1}{f(z)}$ .
  - (c) Describe the image of the vertical line  $\operatorname{Re} z = 1$ .
  - (d) Describe the image of the horizontal line  $\operatorname{Im} z = \pi/4$ .
  - (e) Describe the image of the infinite strip  $0 \leq \operatorname{Im} z \leq \pi/4$ .
  
2. Show that  $w = z^2$  maps
  - (a) the line  $x = 1$  into a parabola.
  - (b) the hyperbola  $xy = 1$  into a straight line.
  - (c) the circle  $|z - 1| = 1$  into the curve  $w = 2(1 + \cos \theta)e^{i\theta}$ .(2 + 3 + 5 points)
  
3. (a) Derive the CR equations in polar form:  $u_r = \frac{1}{r}v_\theta$ ,  $v_r = -\frac{1}{r}u_\theta$ .  
(b) Use (a) to compute the derivative of  $f(z) = \log z$ .  
(8 + 2 points)
  
4. Evaluate  $\int_{\Gamma} \frac{z dz}{(z+2)(z-1)}$  where  $\Gamma$  is the circle  $|z| = 4$  traversed twice in the clockwise direction. (10 points)
  
5. If  $f(z)$  is analytic in a disc  $D(0, R)$ , use the Cauchy integral formula to show that  $f(z)$  has a series expansion  $f(z) = a_0 + a_1z + a_2z^2 + \dots$  (10 points)
  
6. Find Laurent series for  $\frac{z}{(z+1)(z+2)}$  in  $1 < |z| < 2$ . (10 points)
  
7. Show, using Residue Calculus, that  $\lim_{R \rightarrow \infty} \int_{-R}^R \frac{dx}{(x^2 + 2x + 2)} = \pi$ . (10 points)