

Serial No:

Student No.:

Name:

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| <p>1. SHOW ALL WORK. NO CREDITS FOR ANSWERS NOT SUPPORTED BY WORK.
2. CALCULATORS ARE NOT ALLOWED.</p> |
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Problem 1 (30 Points): If the limit exists find it. If it does not exist, say so; use ∞ and $-\infty$ when appropriate.

(a) $\lim_{x \rightarrow 3^+} \sqrt{x^2 - 2x + 1} =$

(b) $\lim_{x \rightarrow 2^+} \frac{x}{x^2 - x - 2} =$

(c) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x} =$

(d) $\lim_{x \rightarrow -\infty} \frac{3x\sqrt{x}}{(1+x)(5-\sqrt{x})} =$

Problem 2 (30 Points):

(a) Use the definition of the derivative to find $f'(x)$ where $f(x) = \sqrt{1-2x}$.

(b) Find all values of A and B which will make the following function continuous.

$$f(x) = \begin{cases} x^2 - 2A & \text{if } x \leq 0 \\ B + 5x & \text{if } 0 < x < 2 \\ A - 2B - 2x & \text{if } 2 \leq x \end{cases}$$

(c) Find all points where the function $f(x) = \frac{x}{x^2 - 2x}$ is discontinuous. Write the type of each one.

Problem 3 (30 Points)

(a) Evaluate the limit if it exists. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x)$

(b) The volume V of a spherical cell is given by $V = \frac{4}{3}\pi r^3$, where r is the radius. Find the rate of change and the percentage rate of change of volume with respect to the radius when $r = 5$ cm.

(c) The average cost \bar{C} of producing q units of a product is given by

$\bar{C} = 0.001q^2 - 0.2q + 15 + \frac{1500}{q}$. Find the marginal cost when $q = 100$ and interpret your result.

Problem 4 (30 Points)

(a) Find the equation of the line tangent to the graph of $y = (1 + \sqrt{x+1})^3$ at the point $(0, 8)$.

(b) If the saving function is $S = 5 + 0.4I - 2.5\sqrt{I}$, find the marginal propensity to consume when $I = 25$.

(c) The demand equation for a product is given by $p = 100 - \sqrt{q^2 + 64}$. Approximate the revenue from selling the 7th unit.

Problem 5 (30 Points)

(a) Find the value(s) of a which will make the function $f(x)$ **differentiable**. *Explain why.*

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ a - 1 & \text{if } x = 3 \end{cases}$$

(b) Find $f'(x)$ where $f(x) = \left(\frac{x^3 + 1}{\sqrt{x - 3}} \right)^5$. Do not simplify the answer.

(c) Suppose that m employees produce a total of q units of product per day where $q = m^2 - 3m$. If the demand equation is $p = -0.2q^2 + 100$, find the marginal revenue and the marginal revenue product when $m = 5$.