No programmable calculators and mobile phones allowed in the examination hall. For all questions show calculations in support of your answers.
Q1 Use first principles to find Laplace transform of $t \sinh t$. 
Q2. Use formulae to find Laplace transform of the following functions:

(a): \[ f(t) = \begin{cases} 
2 & 0 \leq t < 2 \\
-1 & 2 \leq t < 4 \\
0 & t \geq 4 
\end{cases} \]

(b): \[ t^2 \sin t \]
Q3. Evaluate Laplace transform of $\int_0^t \sin \pi \tau \, d\tau$ without evaluating the integral.
Q4. Find inverse Laplace transforms of the following:

(a): \[ \frac{e^{-s}}{s(s + 1)} \]

(b): \[ \frac{1}{(s^2 + 1)^2} \]
Q5. Solve \( y'' + y = 4\delta(t - 3\pi) \), \( y(0) = 1 \), \( y'(0) = 0 \)
Q6. Write formula to evaluate the Fourier series of \( f(x) = \begin{cases} 0 & -3 \leq x < 0 \\ x & 0 \leq x < 3 \end{cases} \). Calculate coefficients \( a_0 \) and \( a_n \) only.
Q7. Find the half range sine expansion of \( f(x) = \begin{cases} \pi & 0 < x < 1/2 \\ 0 & 1/2 \leq x < 1 \end{cases} \)
Q8. Find non trivial Eigen functions of $y'' + 4\lambda y = 0$ subject to $y'(0) = 0, y'(L) = 0$