KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 101 - EXAM II

Sunday - December 9, 2007

Test Code: 1 Duration 120 Minutes

Student’s Name: ..............................................................

ID #: ................................. Section #: ..........................

Important Instructions:

1. All types of CALCULATORS, PAGERS, OR MOBILES ARE NOT ALLOWED to be with you during the examination.

2. Use an HB 2 pencil.

3. Use a good eraser. Do not use the eraser attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Section number, be sure that bubbles match with the number that you write.

6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.

9. Check that the exam paper has 20 questions.
1. \( \lim_{t \to 0} t \cot 2t = \)

(a) 0
(b) 4
(c) \( \frac{1}{2} \)
(d) \( \infty \)
(e) does not exist

2. If \( x^3 - y^3 = 1 \), then \( y''(x) = \)

(a) \( -\frac{2x}{y^4} \)
(b) \( \frac{2x}{y^5} \)
(c) \( \frac{x}{y^5} \)
(d) \( -\frac{2x}{y^5} \)
(e) \( \frac{2x}{y^4} \)
3. If \(1 + xy + y \cos y = e^{1-x} - \frac{\pi}{2}\), then \(y'\) at \((1, -\frac{\pi}{2})\) is equal to

(a) \(\frac{\pi}{2}\)

(b) \(-1\)

(c) \(0\)

(d) \(-2\)

(e) \(1\)

4. If \(y = x^{\tan x}\), then \(y'\left(\frac{\pi}{4}\right) = \)

(a) \(1\)

(b) \(\frac{\pi}{4} \ln \frac{\pi}{4}\)

(c) \(1 + \frac{\pi}{4} \ln \frac{\pi}{4}\)

(d) \(\frac{\pi}{2} \ln \frac{\pi}{4}\)

(e) \(1 + \frac{\pi}{2} \ln \frac{\pi}{4}\)
5. If \( f(x) = x \cos^{-1}(2x) - \frac{1}{2} \sqrt{1 - 4x^2} \), then \( f'(x) = \)

(a) \( 4 \cos^{-1}(2x) \)

(b) \( \cos^{-1}(2x) \)

(c) \( \frac{1}{4} \cos^{-1}(2x) \)

(d) \( \cos^{-1}(2x) - \frac{x}{\sqrt{1 - 4x^2}} \)

(e) \( \frac{\cos^{-1}(2x)}{\sqrt{1 - 4x^2}} \)

6. If \( y = 3^x \cdot x^3 \), then \( y'(1) = \)

(a) 6

(b) \( 9 + 3 \ln 3 \)

(c) 12

(d) \( 9 + \ln 9 \)

(e) \( 3 + 3 \ln 3 \)
7. An equation of the tangent line to the curve $xe^y = y - 1$ at $x = 0$ is given by

(a) $y = e \cdot x$
(b) $y = x + 1$
(c) $y = e \cdot x + 1$
(d) $y = 2e \cdot x + 1$
(e) $y = x$

8. A particle moves in a straight line and its position is given by

$$s(t) = 2t^3 - 12t^2 + 18t + 5$$

feet in $t$ seconds. The total distance traveled by the particle during the first 4 seconds is

(a) $s(1) + s(4) - s(3)$
(b) $2s(1) + s(4) - s(3) - 5$
(c) $2s(1) + s(4) - 2s(3) - 5$
(d) $s(4) - 5$
(e) $2s(1) + s(4) - 5$
9. Suppose that \( F(x) = f(g(x)) \) and \( g(3) = 6, g'(3) = 4, f'(3) = 2 \) and \( f'(6) = 7 \). Then \( F'(3) \) is equal to

(a) 8  
(b) 14  
(c) 24  
(d) 42  
(e) 28

10. If \( y = e^{x^2} \), then \( y''' - 2xy'' - 6y' = \)

(a) 0  
(b) \( 8xe^{x^2} \)  
(c) \( -4xe^{x^2} \)  
(d) \( 8x^3e^{x^2} \)  
(e) \( (12x - 8x^3)e^{x^2} \)
11. If \( f(x) = \tanh^{-1}(\sin x) \), then \( f'(0) = \)

(a) \( \frac{1}{2} \)

(b) \(-1\)

(c) 0

(d) \(-\frac{1}{2}\)

(e) 1

12. If \( g(2x + 1) = \sqrt{x^2 + 8x} \), then \( g'(3) = \)

(a) \( \frac{5}{3} \)

(b) \( \frac{5}{6} \)

(c) \( \frac{7}{2\sqrt{37}} \)

(d) \( \frac{7}{\sqrt{37}} \)

(e) \( \frac{5}{12} \)
13. If \( y = \log_3 \sqrt{x^2 + 1} \), then \( y'(1) = \)

(a) \( \frac{2}{\ln 3} \)  
(b) \( \frac{1}{2} \)  
(c) \( \frac{1}{\ln 3} \)  
(d) \( \ln 3 \)  
(e) \( \frac{1}{2\ln 3} \)

14. Two ships start moving from the same point. One sails north at 60 mi/h and the other sails east at 25 mi/h. At what rate is the distance between the ships increasing two hours later?

(a) 130 mi/h  
(b) 35 mi/h  
(c) 85 mi/h  
(d) 170 mi/h  
(e) 65 mi/h
15. If \( y = \frac{(x^2 - 8)^{1/3} \sqrt{x^3 + 1}}{x^6 - 7x + 5} \), then \( \frac{dy}{dx} \bigg|_{x=0} = \)

(a) \( \frac{14}{5} \)

(b) \( \frac{14}{25} \)

(c) \( -\frac{14}{5} \)

(d) \( -\frac{14}{25} \)

(e) \( -\frac{7}{25} \)

16. If \( y = \tanh(\ln x) \), then \( y'(1) = \)

(a) 2

(b) \( \frac{1}{2} \)

(c) 1

(d) \( \frac{1}{4} \)

(e) 0
17. Which of the following is not an identity:

(a) \( \sinh(x + y) = \sinh x \cdot \cosh y + \cosh x \cdot \sinh y \)

(b) \( \sinh^2 x + \cosh^2 x = 1 \)

(c) \( \cosh(x + y) = \cosh x \cdot \cosh y + \sinh x \cdot \sinh y \)

(d) \( \cosh x + \sinh x = e^x \)

(e) \( \cosh x - \sinh x = e^{-x} \)

18. If \( y = \sin(u^2 - 4) \) and \( u = 2e^x - x \), then \( \frac{dy}{dx} \bigg|_{x=0} = \)

(a) 4

(b) -4

(c) -2

(d) 0

(e) 2
19. If \( f(x) = (1 - x)^{-1} + e^{-2x} \), then \( f^{(100)}(0) =

(a) \(-100! - 2^{100}\)
(b) \(100! - 2^{100}\)
(c) \(100! + (-2)^{100}\)
(d) \(-100! + 2^{100}\)
(e) \(100!\)

20. The volume of a cube is increasing at a rate of 10 cm\(^3\)/min. How fast is the surface area increasing when the length of an edge is 30 cm?

(a) \(120\text{ cm}^2/\text{min}\)
(b) \(\frac{4}{3}\text{ cm}^2/\text{min}\)
(c) \(4\text{ cm}^2/\text{min}\)
(d) \(\frac{2}{9}\text{ cm}^2/\text{min}\)
(e) \(\frac{8}{3}\text{ cm}^2/\text{min}\)